Chapter 1

PRACTICE PROBLEM SET 1

Try these 30 problems to test your skill with limits. The answers are in Chapter 21.

1. $\lim_{x\to 8}\left(x^2-5x-11\right)=$

2. $\lim_{x\to 5}\left(\frac{x+3}{x^2-15}\right)=$

3. $\lim_{x\to 0}\pi^2=$

4. $\lim_{x\to 3}\left(\frac{x^2-2x-3}{x-3}\right)=$

5. $\lim_{x\to\infty}\left(\frac{10x^2+25x+1}{x^4-8}\right)=$

6. $\lim_{x\to\infty}\left(\frac{x^4-8}{10x^2+25x+1}\right)=$

7. $\lim_{x\to\infty}\left(\frac{x^4-8}{10x^4+25x+1}\right)=$

8. $\lim_{x\to\infty}\left(\frac{\sqrt{5x^4+2x}}{x^2}\right)=$

9. $\lim_{x\to 5^+}\left(\frac{x+2}{x^2-4x-12}\right)=$

10. $\lim_{x\to 5^-}\left(\frac{x+2}{x^2-4x-12}\right)=$

11. $\lim_{x\to 6}\left(\frac{x+2}{x^2-4x-12}\right)=$

12. $\lim_{x\to 0^+}\left(\frac{x}{|x|}\right)=$

13. $\lim_{x\to 0^-}\left(\frac{x}{|x|}\right)=$

14. $\lim_{x\to 7^+}\left(\frac{x}{x^2-49}\right)=$

15. $\lim_{x\to 7}\left(\frac{x}{x^2-49}\right)=$

16. $\lim_{x\to 7}\frac{x}{(x-7)^2}=$

17. Let $f(x)=\begin{cases}x^2-5, x\le 3\\ x+2, x>3\end{cases}$

Find: (a) $\lim_{x\to 3^-} f(x)$; (b) $\lim_{x\to 3^+} f(x)$; and (c) $\lim_{x\to 3} f(x)$

18. Let $f(x)=\begin{cases}x^2-5, x\le 3\\ x+1, x>3\end{cases}$

Find: (a) $\lim_{x\to 3^-} f(x)$; (b) $\lim_{x\to 3^+} f(x)$; and (c) $\lim_{x\to 3} f(x)$

19. Find $\lim_{x\to\frac{\pi}{4}} 3\cos x$.

20. Find $\lim_{x\to 0} 3\frac{x}{\cos x}$.

21. Find $\lim_{x\to 0} 3\frac{x}{\sin x}$.

22. Find $\lim_{x\to 0}\frac{\sin 3x}{\sin 8x}$.

23. Find $\lim_{x\to 0}\frac{\tan 7x}{\sin 5x}$.

24. Find $\lim_{x\to\infty}\sin x$.

25. Find $\lim_{x\to\infty}\sin\frac{1}{x}$.

26. Find $\lim_{x\to 0}\frac{x^2\sin x}{1-\cos^2 x}$.

27. Find $\lim_{x\to 0}\frac{\sin^2 7x}{\sin^2 11x}$.

28. Find $\lim_{h\to 0}\frac{(3+h)^2-9}{h}$.

29. Find $\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$.

30. Find $\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$.

Directions: Answer these questions without using your calculator.

$\lim_{x\to 2} \frac{x^2-4}{x^2+4}$ is

(A) 1 (B) 0 (C) $-\frac{1}{2}$ (D) -1 (E) ∞

$\lim_{x\to 2} \frac{4-x^2}{x^2-1}$ is

(A) 1 (B) 0 (C) -4 (D) -1 (E) ∞

$\lim_{x\to 3} \frac{x-3}{x^2+4}$ is

(A) 0 (B) 1 (C) $\frac{1}{4}$ (D) ∞ (E) none of these

$\lim_{x\to 0} \frac{x}{x}$ is

(A) 1 (B) 0 (C) ∞ (D) -1 (E) nonexistent

$\lim_{x\to 2} \frac{x^3-8}{x^2-4}$ is

(A) 4 (B) 0 (C) 1 (D) 3 (E) ∞

$\lim_{x\to \infty} \frac{4-x^2}{4x^2-x-2}$ is

(A) -2 (B) $-\frac{1}{4}$ (C) 1 (D) 2 (E) nonexistent

$\lim_{x\to \infty} \frac{5x^3+27}{20x^2+10x+9}$ is

(A) $-\infty$ (B) -1 (C) 0 (D) 3 (E) ∞

$\lim_{x\to \infty} \frac{3x^2+27}{x^3-27}$ is

(A) 3 (B) ∞ (C) 1 (D) -1 (E) 0

$\lim_{x\to \infty} \frac{2^{-x}}{2^x}$ is

(A) -1 (B) 1 (C) 0 (D) ∞ (E) none of these

$\lim x\to\infty 2-x2x$ is

(A) -1 (B) 1 (C) 0 (D) ∞ (E) none of these

14. The graph of $y = \dfrac{x^2-9}{3x-9}$ has

(A) a vertical asymptote at $x = 3$ (B) a horizontal asymptote at $y = \frac{1}{3}$

(C) a removable discontinuity at $x = 3$ (D) an infinite discontinuity at $x = 3$

17. Which statement is true about the curve $y = \dfrac{2x^2+4}{2+7x-4x^2}$?

(A) The line $x = -\frac{1}{4}$ is a vertical asymptote.

(B) The line $x = 1$ is a vertical asymptote.

(C) The line $y = -\frac{1}{4}$ is a horizontal asymptote.

(D) The graph has no vertical or horizontal asymptote.

(E) The line $y = 2$ is a horizontal asymptote.

18. $\lim\limits_{x\to\infty} \dfrac{2x^2+1}{(2-x)(2+x)}$ is

(A) -4 (B) -2 (C) 1 (D) 2 (E) nonexistent

26. The graph of $y = \dfrac{2x^2+2x+3}{4x^2-4x}$ has

(A) a horizontal asymptote at $y = +\frac{1}{2}$ but no vertical asymptote

(B) no horizontal asymptote but two vertical asymptotes, at $x = 0$ and $x = 1$

(C) a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = 0$ and $x = 1$

(D) a horizontal asymptote at $x = 2$ but no vertical asymptote

(E) a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = \pm 1$

MR CARVER'S LIMIT TIP SHEET

ALWAYS START BY PLUGGING IN c!

IF $\lim_{x\to c} f(x) = \frac{0}{0}$ YOU ARE NOT DONE!

YOU WILL:
* MULTIPLY BY THE CONJUGATE
* COMMON DENOMINATOR & COPY, DOT, FLOP
* SPLIT AND USE TRIG. IDENTITIES
* FACTOR AND CANCEL → $\lim_{x\to 2} \frac{|x-2|}{x-2}$
* THINK OF A SPECIAL CASE

IF $\lim_{x\to c} f(x) = \frac{0}{\text{A NON-ZERO \#}} = 0$ ALWAYS! $\left[\lim_{x\to 3} \frac{x-3}{x^2+9} = \frac{0}{18} = 0\right]$

IF $\lim_{x\to c} f(x) = \frac{\text{A NON-ZERO \#}}{0}$ = * "DNE" IF THE LIMIT IS TWO SIDED
* EITHER ∞ OR $-\infty$ IF THE LIMIT IS TWO SIDED

IF $\lim_{x\to c} f(x) = \frac{\text{A NON ZERO \#}}{\text{A NON ZERO \#}}$ THE LIMIT WILL BE THAT FRACTION

$\left[\lim_{x\to 2} \frac{x^2+2}{x^2-2} = \frac{6}{2} = 3\right]$

PRACTICE PROBLEMS

Now try these problems. The answers are in chapter 21.

1. Is the function $f(x)=\begin{cases}x+7, & x<2\\ 9, & x=2\\ 3x+3, & x>2\end{cases}$ continuous at $x=2$?

2. Is the function $f(x)=\begin{cases}4x^2-2x, & x<3\\ 10x-1, & x=3\\ 30, & x>3\end{cases}$ continuous at $x=3$?

3. Is the function $f(x)=\begin{cases}5x+7, & x<3\\ 7x+1, & x>3\end{cases}$ continuous at $x=3$?

4. Is the function $f(x)=\sec x$ continuous everywhere?

5. Is the function $f(x)=\sec x$ continuous on the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$?

6. Is the function $f(x)=\sec x$ continuous on the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$?

7. For what value(s) of k is the function $f(x)=\begin{cases}3x^2-11x-4, & x\le 4\\ kx^2-2x-1, & x>4\end{cases}$ continuous at $x=4$?

8. For what value(s) of k is the function $f(x)=\begin{cases}-6x-12, & x<-3\\ k^2-5k, & x=-3\\ 6, & x>-3\end{cases}$ continuous at $x=-3$?

9. At what point is the removable discontinuity for the function $f(x)=\dfrac{x^2+5x-24}{x^2-x-6}$?

10. Given the graph of $f(x)$ above, find:

(a) $\lim_{x\to-\infty} f(x)$

(b) $\lim_{x\to\infty} f(x)$

(c) $\lim_{x\to 3^-} f(x)$

(d) $\lim_{x\to 3^+} f(x)$

(e) $f(3)$

(f) Any other discontinuities.