

# Chapter 1

## MR CARVER'S LIMIT TIP SHEET

ALWAYS START BY PLUGGING IN C!

IF  $\lim_{x \rightarrow c} f(x) = \frac{0}{0}$  YOU ARE NOT DONE!

YOU WILL:

- \* MULTIPLY BY THE CONJUGATE
- \* COMMON DENOMINATOR & COPY, DOT, FLOP

- \* SPLIT AND USE TRIG. IDENTITIES

- \* FACTOR AND CANCEL  $\rightarrow \lim_{x \rightarrow 2} \frac{|x-2|}{x-2}$

- \* THINK OF A SPECIAL CASE

IF  $\lim_{x \rightarrow c} f(x) = \frac{0}{\text{A NON-ZERO \#}} = 0$

ALWAYS!

$$\lim_{x \rightarrow 3} \frac{x-3}{x^2+9} = \frac{0}{18} = 0$$

IF  $\lim_{x \rightarrow c} f(x) = \frac{\text{A NON-ZERO \#}}{0} = *$  "ONE" IF THE LIMIT IS TWO SIDED

\* EITHER  $\infty$  OR  $-\infty$  IF THE LIMIT ONE SIDED

IF  $\lim_{x \rightarrow c} f(x) = \frac{\text{A NON-ZERO \#}}{\text{A NON-ZERO \#}}$

THE LIMIT WILL BE THAT FRACTION

$$\lim_{x \rightarrow 2} \frac{x^2+2}{x^2-2} = \frac{6}{2} = 3$$

**PRACTICE PROBLEM SET 1**

Try these 30 problems to test your skill with limits. The answers are in Chapter 21.

1.  $\lim_{x \rightarrow 8} (x^2 - 5x - 11) =$

2.  $\lim_{x \rightarrow 5} \left( \frac{x+3}{x^2-15} \right) =$

3.  $\lim_{x \rightarrow 0} \pi^2 =$

4.  $\lim_{x \rightarrow 3} \left( \frac{x^2 - 2x - 3}{x - 3} \right) =$

5.  $\lim_{x \rightarrow \infty} \left( \frac{10x^2 + 25x + 1}{x^4 - 8} \right) =$

6.  $\lim_{x \rightarrow \infty} \left( \frac{x^4 - 8}{10x^2 + 25x + 1} \right) =$

7.  $\lim_{x \rightarrow \infty} \left( \frac{x^4 - 8}{10x^4 + 25x + 1} \right) =$

8.  $\lim_{x \rightarrow \infty} \left( \frac{\sqrt{5x^4 + 2x}}{x^2} \right) =$

9.  $\lim_{x \rightarrow 6^+} \left( \frac{x+2}{x^2 - 4x - 12} \right) =$

10.  $\lim_{x \rightarrow 6^-} \left( \frac{x+2}{x^2 - 4x - 12} \right) =$

11.  $\lim_{x \rightarrow 6} \left( \frac{x+2}{x^2 - 4x - 12} \right) =$

12.  $\lim_{x \rightarrow 0^+} \left( \frac{x}{|x|} \right) =$

13.  $\lim_{x \rightarrow 0^-} \left( \frac{x}{|x|} \right) =$

14.  $\lim_{x \rightarrow 7^+} \left( \frac{x}{x^2 - 49} \right) =$

15.  $\lim_{x \rightarrow 7^-} \left( \frac{x}{x^2 - 49} \right) =$

16.  $\lim_{x \rightarrow 7} \frac{x}{x^2 - 49} =$

17. Let  $f(x) = \begin{cases} x^2 - 5, & x \leq 3 \\ x + 2, & x > 3 \end{cases}$

Find: (a)  $\lim_{x \rightarrow 3^-} f(x)$ ; (b)  $\lim_{x \rightarrow 3^+} f(x)$ ; and (c)  $\lim_{x \rightarrow 3} f(x)$

18. Let  $f(x) = \begin{cases} x^2 - 5, & x \leq 3 \\ x + 1, & x > 3 \end{cases}$

Find: (a)  $\lim_{x \rightarrow 3^-} f(x)$ ; (b)  $\lim_{x \rightarrow 3^+} f(x)$ ; and (c)  $\lim_{x \rightarrow 3} f(x)$

19. Find  $\lim_{x \rightarrow \frac{\pi}{4}} 3 \cos x$ .

20. Find  $\lim_{x \rightarrow 0} 3 \frac{x}{\cos x}$ .

21. Find  $\lim_{x \rightarrow 0} 3 \frac{x}{\sin x}$ .

22. Find  $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 8x}$ .

23. Find  $\lim_{x \rightarrow 0} \frac{\tan 7x}{\sin 5x}$ .

24. Find  $\lim_{x \rightarrow \infty} \sin x$ .

25. Find  $\lim_{x \rightarrow \infty} \sin \frac{1}{x}$ .

26. Find  $\lim_{x \rightarrow 0} \frac{x^2 \sin x}{1 - \cos^2 x}$ .

27. Find  $\lim_{x \rightarrow 0} \frac{\sin^2 7x}{\sin^2 11x}$ .

28. Find  $\lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{h}$ .

29. Find  $\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$ .

30. Find  $\lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$ .

## More Limits... Find Them

Show work which supports your answer.

1.  $\lim_{x \rightarrow 16^+} \frac{x-16}{\sqrt{x}-4}$

2.  $\lim_{x \rightarrow 1} \left( \frac{x^2}{x-1} - \frac{1}{x-1} \right)$

3.  $\lim_{x \rightarrow -3} \frac{x+3}{\frac{1}{1} \frac{1}{x+3}}$

4.  $\lim_{x \rightarrow 2} \frac{|x-2|}{x-2}$

5.  $\lim_{x \rightarrow \pi^-} \frac{|\pi-x|}{x-\pi}$

$$6. \lim_{x \rightarrow 3} \frac{5x}{6-2x}$$

$$7. \lim_{x \rightarrow 3^+} \frac{x^2+x+2}{x^2-2x-3}$$

$$8. \lim_{x \rightarrow 3^-} \frac{x^2+x+2}{x^2-2x-3}$$

$$9. \lim_{x \rightarrow 0^+} x \sqrt{1 + \frac{1}{x^2}}$$

$$10. \lim_{x \rightarrow 0^-} x \sqrt{1 + \frac{1}{x^2}}$$

## PRACTICE PROBLEMS

Now try these problems. The answers are in chapter 21.

1. Is the function  $f(x) = \begin{cases} x+7, & x < 2 \\ 9, & x = 2 \\ 3x+3, & x > 2 \end{cases}$  continuous at  $x = 2$ ?

2. Is the function  $f(x) = \begin{cases} 4x^2 - 2x, & x < 3 \\ 10x - 1, & x = 3 \\ 30, & x > 3 \end{cases}$  continuous at  $x = 3$ ?

3. Is the function  $f(x) = \begin{cases} 5x+7, & x < 3 \\ 7x+1, & x > 3 \end{cases}$  continuous at  $x = 3$ ?

4. Is the function  $f(x) = \sec x$  continuous everywhere?

5. Is the function  $f(x) = \sec x$  continuous on the interval  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ?

6. Is the function  $f(x) = \sec x$  continuous on the interval  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ ?

7. For what value(s) of  $k$  is the function  $f(x) = \begin{cases} 3x^2 - 11x - 4, & x \leq 4 \\ kx^2 - 2x - 1, & x > 4 \end{cases}$  continuous at  $x = 4$ ?

8. For what value(s) of  $k$  is the function  $f(x) = \begin{cases} -6x - 12, & x < -3 \\ k^2 - 5k, & x = -3 \\ 6, & x > -3 \end{cases}$  continuous at  $x = -3$ ?

9. At what point is the removable discontinuity for the function  $f(x) = \frac{x^2 + 5x - 24}{x^2 - x - 6}$ ?

10. Given the graph of  $f(x)$  above, find:

(a)  $\lim_{x \rightarrow -\infty} f(x)$

(b)  $\lim_{x \rightarrow \infty} f(x)$

(c)  $\lim_{x \rightarrow 3^-} f(x)$

(d)  $\lim_{x \rightarrow 3^+} f(x)$

(e)  $f(3)$

(f) Any other discontinuities.

