

# PRACTICE PROBLEMS

Now try these problems. The answers are in chapter 21.

KEY

1. Is the function  $f(x) = \begin{cases} x+7, & x < 2 \\ 9, & x = 2 \\ 3x+3, & x > 2 \end{cases}$  continuous at  $x = 2$ ? yes!

2. Is the function  $f(x) = \begin{cases} 4x^2 - 2x, & x < 3 \\ 10x - 1, & x = 3 \\ 30, & x > 3 \end{cases}$  continuous at  $x = 3$ ? NO! FAILS #3

3. Is the function  $f(x) = \begin{cases} 5x+7, & x < 3 \\ 7x+1, & x > 3 \end{cases}$  continuous at  $x = 3$ ? NO! FAILS #1

4. Is the function  $f(x) = \sec x$  continuous everywhere? NO! DISC @  $\frac{\pi}{2} + \pi k$

5. Is the function  $f(x) = \sec x$  continuous on the interval  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ ? NO DISC @ endpoints

6. Is the function  $f(x) = \sec x$  continuous on the interval  $(-\frac{\pi}{2}, \frac{\pi}{2})$ ? yes!

7. For what value(s) of  $k$  is the function  $f(x) = \begin{cases} 3x^2 - 11x - 4, & x \leq 4 \\ kx^2 - 2x - 1, & x > 4 \end{cases}$  continuous at  $x = 4$ ?  $k = \frac{9}{16}$

8. For what value(s) of  $k$  is the function  $f(x) = \begin{cases} -6x - 12, & x < -3 \\ k^2 - 5k, & x = -3 \\ 6, & x > -3 \end{cases}$  continuous at  $x = -3$ ?  $k = 6, -1$

9. At what point is the removable discontinuity for the function  $f(x) = \frac{x^2 + 5x - 24}{x^2 - x - 6}$ ?  $(3, \frac{11}{5})$

10. Given the graph of  $f(x)$  above, find:

(a)  $\lim_{x \rightarrow -\infty} f(x) = 0$

(b)  $\lim_{x \rightarrow \infty} f(x) = 0$

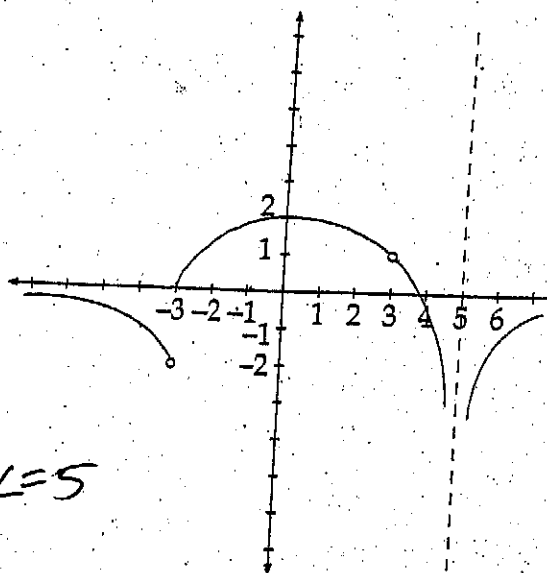
(c)  $\lim_{x \rightarrow 3^-} f(x) = 1$

(d)  $\lim_{x \rightarrow 3^+} f(x) = 1$

(e)  $f(3)$  DNE

(f) Any other discontinuities.

@  $x = -3, x = 3, x = 5$



# PRACTICE PROBLEMS

Try these 30 problems to test your skill with limits. The answers are in chap

KEY

1.  $\lim_{x \rightarrow 8} (x^2 - 5x - 11) = 13$

2.  $\lim_{x \rightarrow 3} \left( \frac{x+3}{x^2-15} \right) = 8/10$

3.  $\lim_{x \rightarrow 0} \pi^2 = \pi^2$

4.  $\lim_{x \rightarrow 3} \left( \frac{x^2 - 2x - 3}{x - 3} \right) = 4$

5.  $\lim_{x \rightarrow -1} \left( \frac{10x^2 + 25x + 1}{x^4 - 8} \right) = 0$

6.  $\lim_{x \rightarrow -1} \left( \frac{x^4 - 8}{10x^2 + 25x + 1} \right) = \infty$

7.  $\lim_{x \rightarrow -1} \left( \frac{x^4 - 8}{10x^4 + 25x + 1} \right) = 1/10$

8.  $\lim_{x \rightarrow \infty} \left( \frac{\sqrt{5x^4 + 2x}}{x^2} \right) = \sqrt{5}$

9.  $\lim_{x \rightarrow 6^+} \left( \frac{x+2}{x^2 - 4x - 12} \right) = \infty$

10.  $\lim_{x \rightarrow 6^-} \left( \frac{x+2}{x^2 - 4x - 12} \right) = -\infty$

11.  $\lim_{x \rightarrow 6} \left( \frac{x+2}{x^2 - 4x - 12} \right) = \text{DNE}$

12.  $\lim_{x \rightarrow 0^+} \left( \frac{x}{|x|} \right) = 1$

13.  $\lim_{x \rightarrow 0^-} \left( \frac{x}{|x|} \right) = -1$

14.  $\lim_{x \rightarrow 7^+} \left( \frac{x}{x^2 - 49} \right) = \infty$

15.  $\lim_{x \rightarrow 7^-} \left( \frac{x}{x^2 - 49} \right) = \text{DNE}$

16.  $\lim_{x \rightarrow 7} \frac{x}{(x-7)^2} = \text{DNE}$

17. Let  $f(x) = \begin{cases} x^2 - 5, & x \leq 3 \\ x + 2, & x > 3 \end{cases}$

Find: (a)  $\lim_{x \rightarrow 3^-} f(x)$ ; (b)  $\lim_{x \rightarrow 3^+} f(x)$ ; and (c)  $\lim_{x \rightarrow 3} f(x)$

18. Let  $f(x) = \begin{cases} x^2 - 5, & x \leq 3 \\ x + 1, & x > 3 \end{cases}$

Find: (a)  $\lim_{x \rightarrow 3^-} f(x)$ ; (b)  $\lim_{x \rightarrow 3^+} f(x)$ ; and (c)  $\lim_{x \rightarrow 3} f(x)$

19. Find  $\lim_{x \rightarrow \frac{\pi}{4}} 3 \cos x$

20. Find  $\lim_{x \rightarrow 0} 3 \frac{x}{\cos x}$

21. Find  $\lim_{x \rightarrow 0} 3 \frac{x}{\sin x}$

22. Find  $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 8x}$

23. Find  $\lim_{x \rightarrow 0} \frac{\tan 7x}{\sin 5x}$

24. Find  $\lim_{x \rightarrow \pi} \sin x$

25. Find  $\lim_{x \rightarrow \infty} \frac{1}{x}$

26. Find  $\lim_{x \rightarrow 0} \frac{x^2 \sin x}{1 - \cos^2 x}$

27. Find  $\lim_{x \rightarrow 0} \frac{\sin^2 7x}{\sin^2 11x}$

28. Find  $\lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{h}$

29. Find  $\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$

30. Find  $\lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$

$3/\sqrt{2}$