EXTRA PS REV. KEY

1) $f(x) = x^3 \tan^{-1}(x^2)$

if $g(x) = \tan^{-1}(x^2)$, then $f(x) = x^3 g(x)$

to find $g(x)$, find $g'(x) = \frac{1}{1+x^4}(2x) = \frac{2x}{1+x^4}$

so $g'(x) = \sum_{n=0}^{\infty} 2x(-x^4)^n = 2\sum_{n=0}^{\infty} (-1)^n x^{4n+1}$

$g(x) = \int g'(x)\,dx = \int 2\sum_{n=0}^{\infty} (-1)^n x^{4n+1}\,dx$

$g(x) = 2\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{4n+2} = \cancel{2}\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{\cancel{2}(2n+1)}$

$g(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{2n+1}$, then $x^3 g(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^3 x^{4n+2}}{2n+1}$

$$= \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+5}}{2n+1}}$$

2) $f(x) = \ln(2x^3+1)$

Find $f'(x) = \dfrac{1}{2x^3+1}\,6x^2 = \dfrac{6x^2}{1+2x^3}$

so $f'(x) = \displaystyle\sum_{n=0}^{\infty} 6x^2(-2x^3)^n = 6\sum_{n=0}^{\infty}(-1)^n 2^n x^{3n+2}$

$f(x) = \displaystyle\int f'(x)\,dx = \int 6\sum_{n=0}^{\infty}(-1)^n 2^n x^{3n+2}\,dx$

$$= \boxed{6\sum_{n=0}^{\infty}\frac{(-1)^n 2^n x^{3n+3}}{3n+3}}$$

3.) $f(x) = \dfrac{\ln(1+x^4)}{x^4}$

Let $g(x) = \ln(1+x^4)$, then $f(x) = \dfrac{g(x)}{x^4}$

and $g'(x) = \dfrac{4x^3}{1+x^4} = \sum_{n=0}^{\infty} 4x^3(-x^4)^n = 4\sum_{n=0}^{\infty}(-1)^n x^{4n+3}$

and $g(x) = \int 4\sum_{n=0}^{\infty}(-1)^n x^{4n+3} = 4\sum_{n=0}^{\infty}\dfrac{(-1)^n x^{4n+4}}{4n+4} = \sum_{n=0}^{\infty}\dfrac{(-1)^n x^{4n+4}}{n+1}$

$f(x) = \frac{1}{x^4} g(x)$ or $\sum_{n=0}^{\infty}\dfrac{(-1)^n x^{4n+4}}{x^4(n+1)} = \boxed{\sum_{n=0}^{\infty}\dfrac{(-1)^n x^{4n}}{n+1}}$

4.) $f(x) = x^2 \sin(x^3)$ if $g(x) = \sin x = \sum_{n=0}^{\infty}\dfrac{(-1)^n x^{2n+1}}{(2n+1)!}$

and $g(x^3) = \sin(x^3) = \sum_{n=0}^{\infty}\dfrac{(-1)^n (x^3)^{2n+1}}{(2n+1)!} =$

$\sum_{n=0}^{\infty}\dfrac{(-1)^n x^{6n+3}}{(2n+1)!}$

If $f(x) = x^2 g(x^3)$, then $f(x) = \sum_{n=0}^{\infty}\dfrac{(-1)^n x^2 x^{6n+3}}{(2n+1)!}$

$\boxed{= \sum_{n=0}^{\infty}\dfrac{(-1)^n x^{6n+5}}{(2n+1)!}}$

5) $f(x) = \dfrac{\cos(2x^2)}{x^3}$ Let $g(x) = \cos x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n}}{(2n)!}$

then $g(2x^2) = \sum_{n=0}^{\infty} \dfrac{(-1)^n (2x^2)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \dfrac{(-1)^n 2^{2n} x^{4n}}{(2n)!}$

and $f(x) = \dfrac{g(2x^2)}{x^3} = \sum_{n=0}^{\infty} \dfrac{(-1)^n 2^{2n} x^{4n}}{x^3 (2n)!}$

$$\boxed{= \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{4n-3}}{(2n)!}}$$