KEY

Related Rate Supplement

1. Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 4cm/min. How fast is the area of the pool increasing when the radius is 5 cm?

G: $\frac{dr}{dt} = 4\text{cm/min}$ $\quad A = \pi r^2$

W: $\frac{dA}{dt}$ when $r=5$ $\quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(5)(4\text{cm})$

$40\pi\ \text{cm}^2/\text{min}$

2. Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The area of the spill increases at a rate of 90 m²/min. How fast is the radius of the spill increasing when the radius is 10 m?

$\frac{dA}{dt} = 90\text{m}^2/\text{min}$ $\quad A = \pi r^2$

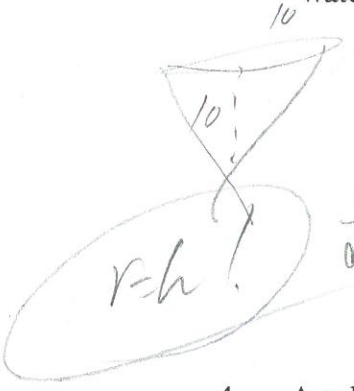
$\frac{dr}{dt} = ?$ when $r=10$ $\quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$90 = 2\pi(10)\frac{dr}{dt}$

$1.43\ \text{m/min}$

$= \frac{9}{2\pi}\ \text{m/min}$

3. A conical paper cup is 10 cm tall with a radius of 10 cm. The cup is being filled with water so that the water level rises at a rate of 2 cm/sec. At what rate is water being poured into the cup when the water level is 8 cm?



$\frac{dh}{dt} = 2\text{cm/sec}$ $\quad V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = ?$ $\quad V = \frac{1}{3}\pi h^2 h$

$V = \frac{1}{3}\pi h^3$

$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \pi(8)^2(2)$

$128\pi\ \text{cm}^3/\text{sec}$

4. A spherical balloon is inflated so that its radius (r) increases at a rate of 2 cm/sec. How fast is the volume of the balloon increasing when the radius is 4 cm?

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = ?$ when $r = 4\text{cm}$

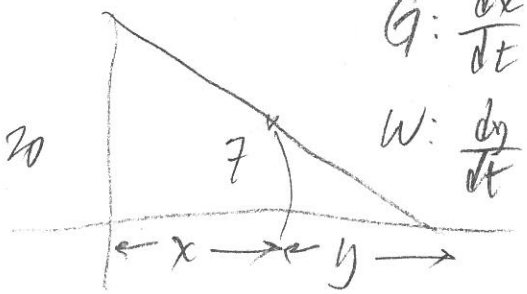
$\frac{dr}{dt} = 2\text{cm/sec}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi(4)^2 2$

$\frac{dV}{dt} = 128\pi\ \text{cm}^3/\text{sec}$

5. A 7 ft tall person is walking away from a 20 ft tall lamppost at a rate of 5 ft/sec. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 16 ft from the lamppost?



G: $\frac{dx}{dt} = 5$ fps

W: $\frac{dy}{dt}$ when $x = 16$

$\frac{20}{x+y} = \frac{7}{y}$

$20y = 7x + 7y$

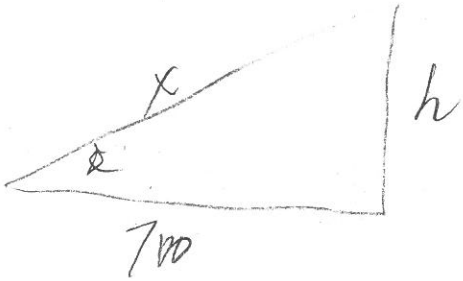
$13y = 7x$

$13\frac{dy}{dt} = 7\frac{dx}{dt}$

$13\frac{dy}{dt} = 7(5)$

$\frac{dy}{dt} = \frac{35}{13}$ ft/sec

6. An observer stands 700 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 900 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 2400 ft from the ground? How fast is the angle of elevation from the observer to the rocket changing when the rocket is 2700 feet above the ground?



G: $\frac{dh}{dt} = 900$ ft/sec

W: $\frac{dx}{dt}$ when $h = 2400$

$700^2 + h^2 = x^2$

$2h\frac{dh}{dt} = 2x\frac{dx}{dt}$

$2(2400)(900) = 2(2500)\frac{dx}{dt}$

$\frac{dx}{dt} = 864$ ft/sec

$\tan\theta = \frac{h}{700}$

$\tan\theta = \frac{2700}{700}$

$\theta = 1.32$

$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{700}\frac{dh}{dt}$

$\frac{d\theta}{dt} = \frac{1}{700}(900)\cos^2\theta$

$\frac{d\theta}{dt} = .081$ rad/sec