PRACTICE PROBLEM SET 26

Calculate the volumes below. The answers are in Chapter 21.

1. Find the volume of the solid that results when the region bounded by $y = \sqrt{9 - x^2}$ and the x-axis is revolved around the x-axis.

2. Find the volume of the solid that results when the region bounded by $y = \sec x$ and the x-axis from $x = -\frac{\pi}{4}$ to $x = \frac{\pi}{4}$ is revolved around the x-axis.

3. Find the volume of the solid that results when the region bounded by $x = 1 - y^2$ and the y-axis is revolved around the y-axis.

4. Find the volume of the solid that results when the region bounded by $x = \sqrt{5}y^2$ and the y-axis from $y = -1$ to $y = 1$ is revolved around the y-axis.

5. Find the volume of the solid that results when the region bounded by $y = x^3$, $x = 2$, and the x-axis is revolved around the line $x = 2$.

6. Use the method of cylindrical shells to find the volume of the solid that results when the region bounded by $y = x$, $x = 2$, and $y = -\frac{x}{2}$ is revolved around the y-axis.

7. Use the method of cylindrical shells to find the volume of the solid that results when the region bounded by $y = \sqrt{x}$, $y = 2x - 1$, and $x = 0$ is revolved around the y-axis.

8. Use the method of cylindrical shells to find the volume of the solid that results when the region bounded by $y = x^2$, $y = 4$, and $x = 0$ is revolved around the x-axis.

9. Use the method of cylindrical shells to find the volume of the solid that results when the region bounded by $y = 2\sqrt{x}$, $x = 4$, and $y = 0$ is revolved around the y-axis.

10. Use the method of cylindrical shells to find the volume of the solid that results when the region bounded by $y^2 = 8x$ and $x = 2$ is revolved around the line $x = 4$.

11. Find the volume of the solid whose base is the region between the semi-circle $y = \sqrt{16 - x^2}$ and the x-axis, and whose cross-sections perpendicular to the x-axis are squares with a side on the base.

12. Find the volume of the solid whose base is the region between $y = x^2$ and $y = 4$ and whose perpendicular cross-sections are isosceles right triangles with the hypotenuse on the base.

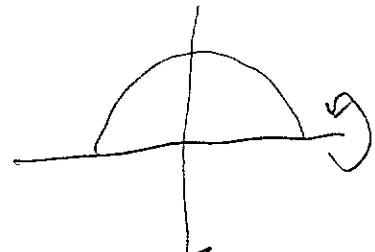
P.S. 26

1–5 DISC/WASHER
6–10 Shell

1) $y = \sqrt{9-x^2}$

$y^2 = 9 - x^2$

$y^2 + x^2 = 9$

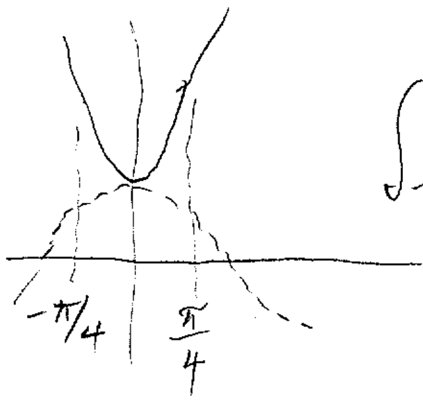


$\pi \int_{-3}^{3} \left(\sqrt{9-x^2}\right)^2 dx = \pi \left[9x - \frac{1}{3}x^3 \right]_{-3}^{3}$

$\pi \left[(27-9) - (-27+9) \right]$

$\pi \left[18 - -18 \right] = 36\pi$

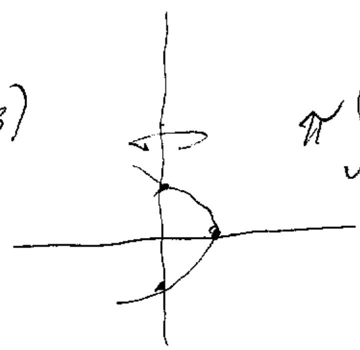
2)



$\int_{-\pi/4}^{\pi/4} (\sec x)^2 dx$

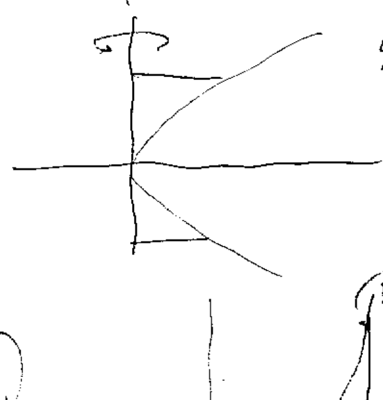
$\pi \tan x \Big]_{-\pi/4}^{\pi/4} = 1 - -1 = 2\pi$

3)



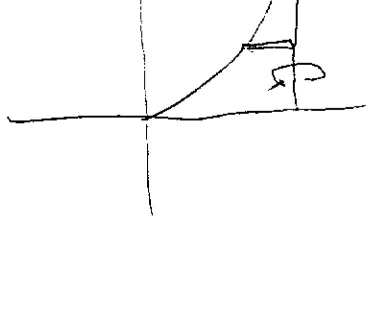
$\pi \int_{-1}^{1} (1-y^2)^2 dy$

4)



$\pi \int_{-1}^{1} (\sqrt{5}\, y^2)^2 dy$

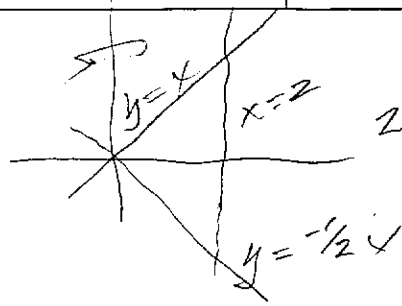
5)



$\pi \int_0^8 (2-x)^2 dy$

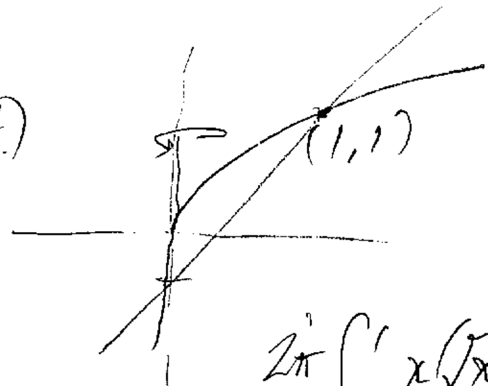
$\pi \int_0^8 (2 - \sqrt[3]{y})^2 dy$

6.)



$$2\pi \int_0^2 x\left(x - \left(-\tfrac{1}{2}x\right)\right) dx$$

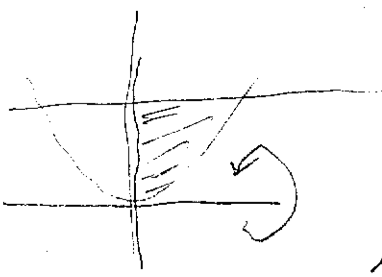
7.)



$$x = 4x^2 - 4x + 1$$
$$4x^2 - 5x + 1$$
$$(4x-1)(x-1)$$
$$x = 1/4$$

$$2\pi \int_0^1 x\left(\sqrt{x} - (2x-1)\right) dx$$

8)

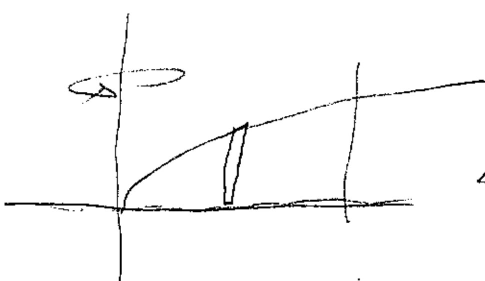


Q1 ONLY?

$y = x^2$

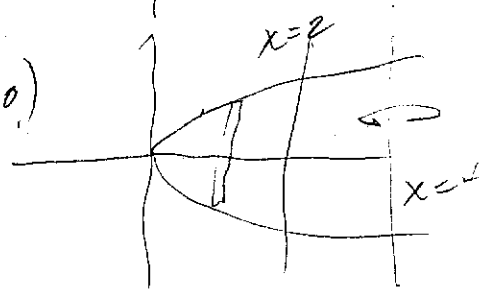
$$2\pi \int_0^4 y\left(\sqrt{y} \quad \right) dy$$

9)



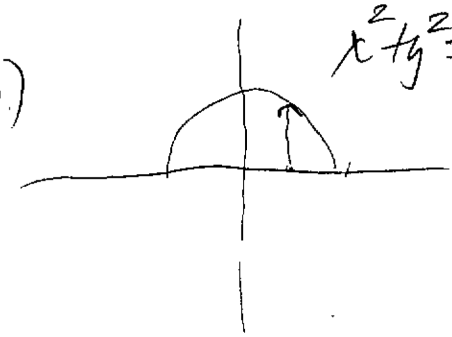
$$2\pi \int_0^4 x\left(2\sqrt{x}\right) dx$$

10.)



$$2\pi \int_0^2 (4-x)\, 2\left(\sqrt{8x}\right) dx$$

11) $x^2+y^2=16$



$$\int_{-4}^{4}\left(\sqrt{16-x^2}\right)^2 dx$$

$$\int_{-4}^{4} 16-x^2\, dx$$

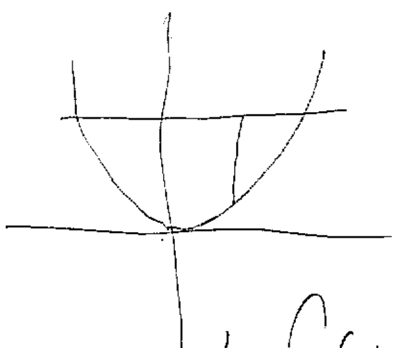
$$\left[16x-\frac{1}{3}x^3\right]_{-4}^{4}$$

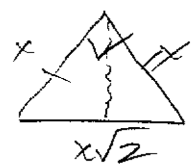
$$\left[64-\frac{64}{3}\right)-\left(-64+\frac{64}{3}\right)$$

$$\left[128-\frac{128}{3}\right]$$

$$\frac{(256)}{3}=$$

12)





$\frac{1}{2}bh\,dx$

$$\frac{1}{2}\int(4-x^2)\left(\frac{4-x^2}{2}\right)dx$$

$$\frac{1}{4}\int 16-8x^2+x^4\,dx$$

$$\frac{1}{4}\left[16x-\frac{8}{3}x^3+\frac{1}{5}x^5\right]_{-2}^{2}$$

$$\frac{1}{4}\left[\left(32-\frac{64}{3}+\frac{32}{5}\right)-\frac{1}{4}\left[-32+\frac{64}{3}-\frac{32}{5}\right]\right.$$