Next, find where the two curves intersect. By setting $y^3 - y = 0$, you'll find that they intersect at $y = -1$, $y = 0$, and $y = 1$. Notice that the curve is to the right of the y-axis from $y = -1$ to $y = 0$ and to the left of the y-axis from $y = 0$ to $y = 1$. Thus, the region must be divided into two parts: from $y = -1$ to $y = 0$ and from $y = 0$ to $y = 1$.

Set up the two integrals:

$$\int_{-1}^{0}\left(y^3 - y\right)dy + \int_{0}^{1}\left(y - y^3\right)dy$$

And integrate them:

$$\left(\frac{y^4}{4} - \frac{y^2}{2}\right)\Bigg|_{-1}^{0} + \left(\frac{y^2}{2} - \frac{y^4}{4}\right)\Bigg|_{0}^{1} = \frac{1}{2}$$

PRACTICE PROBLEM SET 25

Find the area of the region between the two curves in each problem, and be sure to sketch each one. (We only gave you endpoints in one of them.) The answers are in Chapter 21.

1. The curve $y = x^2 - 2$ and the line $y = 2$. $32/3$ $\int_{-2}^{2} 2-(x^2-2)\,dx$

2. The curve $y = x^2$ and the curve $y = 4x - x^2$. $8/3$ $\int_{0}^{2}(4x-x^2-x^2)\,dx$

3. The curve $y = x^3$ and the curve $y = 3x^2 - 4$. $27/4$ $\int_{-1}^{2} x^3-(3x^2-4))\,dx$

4. The curve $y = x^2 - 4x - 5$ and the curve $y = 2x - 5$. 36 $\int_{0}^{4}((2x-5)-(x^2-4x-5))\,dx$

c 5. The curve $y = x^3$ and the x-axis, from $x = -1$ to $x = 2$. $17/4$ $\int_{-1}^{0}(0-x^3)\,dx + \int_{0}^{2} x^3-0\,dx$

6. The curve $x = y^2$ and the line $x = y + 2$. $9/2$ $\int_{-1}^{2} y+2-y^2\,dy$

7. The curve $x = y^2$ and the curve $x = 3 - 2y^2$. 4 $\int_{-1}^{1} 3-2y^2-y^2\,dy$

c 8. The curve $x = y^3 - y^2$ and the line $x = 2y$. $37/12$ $\int_{-1}^{0} y^3-y^2-2y\,dy + \int_{0}^{2} 2-(y^3-y^2)\,dy$

9. The curve $x = y^2 - 4y + 2$ and the line $x = y - 2$. $9/2$ $\int_{1}^{4}(y-2)-(y^2-4y+2)\,dy$

10. The curve $x = y^{\frac{2}{3}}$ and the curve $x = 2 - y^4$. $12/5$

$\int_{-1}^{1} 2-y^4-y^{3/2}\,dy$