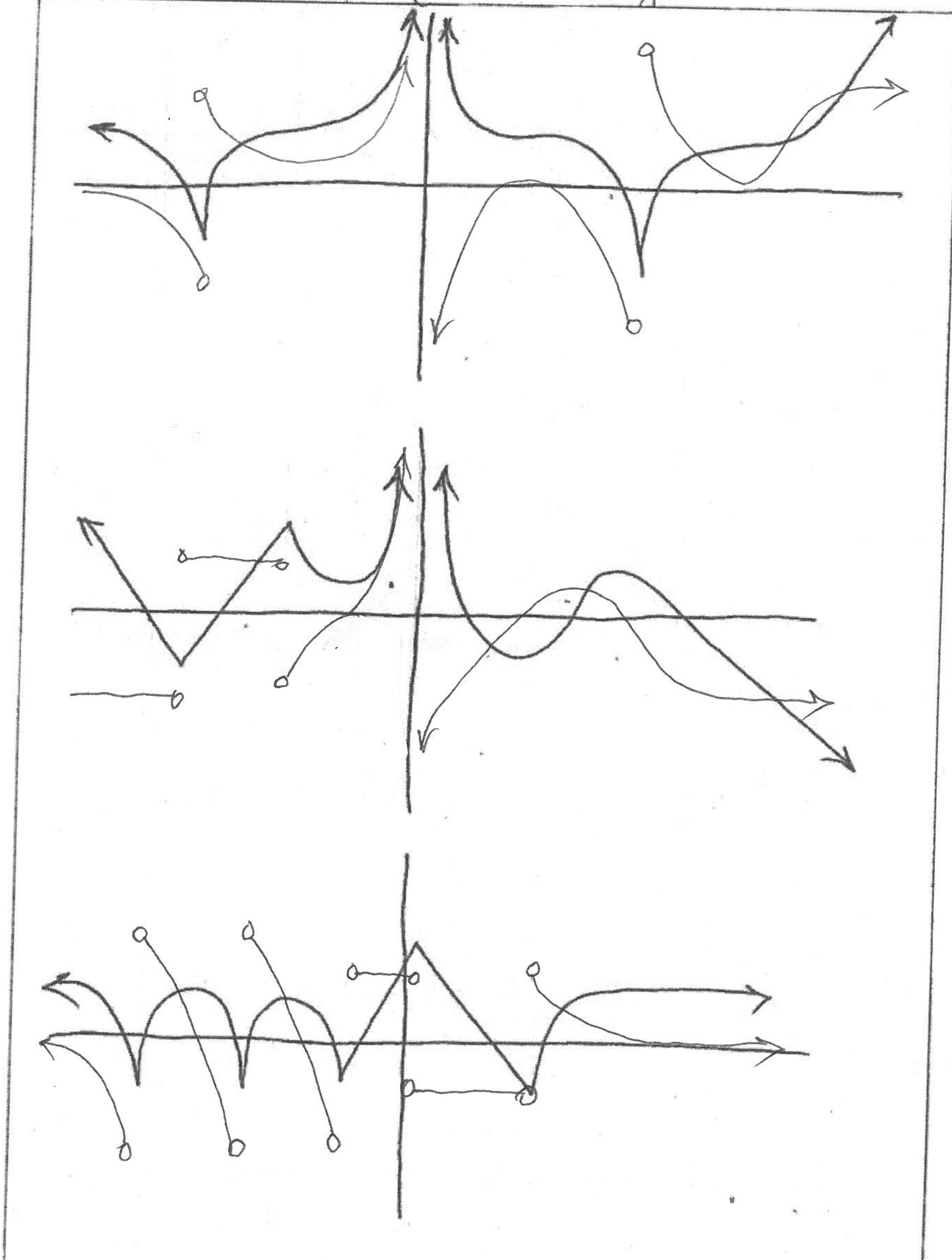
SKETCH $f'(x)$	$(f(x)$ Shown$)$



WB p.35

Projectile Motion

1. A window-cleaning squeegee was dropped from a window-cleaning scaffold platform 1,200 feet above the sidewalk.
$-16t^2 + 1200$

   a. How many seconds will it take the squeegee to hit the ground?

   t = 8.66 seconds

   b. What is the sponge's velocity JUST BEFORE it strikes the ground?

   v(t) = -32t   -277.12 fps

   c. How many seconds will it take to fall 500 feet and how fast will it be traveling?

   t = ~~total~~ sec's 5.59   ~~211.66 fps~~   -178.8 fps

2. After dropping the squeegee from problem #1 the window cleaner trips, which causes his sponge to fly upward at a velocity of 64 feet per second from the platform (1,200 feet above the ground).
$y = -16t^2 + 64t + 1200$

   a. How many seconds will it take the sponge to hit the ground?

   10.89 seconds

   b. How long will it take the sponge to return to the same level as the platform?

   4 seconds

   c. How high will the squeegee be when its velocity is +20feet per second?

$20 = -32t + 64$ $\boxed{1.375}$ t= $\boxed{1257.75' \text{ above ground}}$

3. Once the window cleaner's supplies have either fallen or been tossed, the dejected window cleaner picks up his bucket and throws it downward at a velocity of 80 feet per second toward the bed of his pickup truck parked on the street below (1,195' below the scaffold).

$y = -16t^2 - 80t + 1195$

$-32t - 80$

   a. How many seconds will it take the bucket to hit the bed of the truck?

   t = 6.5 seconds

   b. How long will it take the bucket to be 100 feet above the truck's bed?

   t = 6.14 seconds

   c. What will the bucket's velocity be when it is five feet above the truck's bed?

   t = 6.48   v(6.48) = -287.36 fps.

$0 = -16t^2 + 0t + 5$

4 Later in the day a worker drops a paint brush which hits the ground 7 seconds later - how high

h = ~~784~~ 829.44

Key

Related Rate Supplement

1. Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 4cm/min. How fast is the area of the pool increasing when the radius is 5 cm?

G: $\frac{dr}{dt} = 4\text{ cm/min}$ $\quad A = \pi r^2$

W: $\frac{dA}{dt}$ when $r=5$ $\quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(5)(4\text{ cm})$

$\boxed{40\pi\text{ cm}^2/\text{min}}$

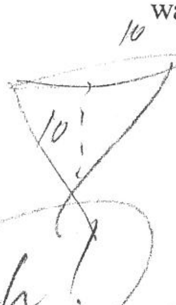
2. Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The area of the spill increases at a rate of 90 m²/min. How fast is the radius of the spill increasing when the radius is 10 m?

$\frac{dA}{dt} = 90\text{ m}^2/\text{min}$ $\quad A = \pi r^2$

$\frac{dr}{dt} = ?$ when $r = 10$ $\quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$90 = 2\pi(10)\frac{dr}{dt}$ $\quad \boxed{= 4.5\pi\text{ m/min}}$

3. A conical paper cup is 10 cm tall with a radius of 10 cm. The cup is being filled with water so that the water level rises at a rate of 2 cm/sec. At what rate is water being poured into the cup when the water level is 8 cm?



$\frac{dh}{dt} = 2\text{ cm/sec}$ $\quad V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = ?$ $\quad V = \frac{1}{3}\pi h^2 h$

$r = h!$

$V = \frac{1}{3}\pi h^3$ $\quad \frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \pi(8)^2(2)$

$\boxed{128\pi\text{ cm/sec}}$

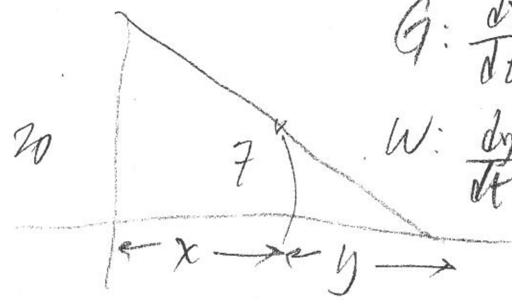
4. A spherical balloon is inflated so that its radius (r) increases at a rate of 2 cm/sec. How fast is the volume of the balloon increasing when the radius is 4 cm?

$V = \frac{4}{3}\pi r^3$ $\quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = ?$ when $r = 4\text{ cm}$ $\quad \frac{dV}{dt} = 4\pi(4)^2 2$

$\frac{dr}{dt} = 2\text{ cm/sec}$ $\quad \frac{dV}{dt} = \boxed{128\pi\text{ cm}^3/\text{sec}}$

5. A 7 ft tall person is walking away from a 20 ft tall lamppost at a rate of 5 ft/sec. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 16 ft from the lamppost?



$G: \frac{dx}{dt} = 7 \text{ fps}$

$W: \frac{dy}{dt}$ when $x = 16$

$\frac{20}{x+y} = \frac{7}{y}$

$20y = 7x + 7y$

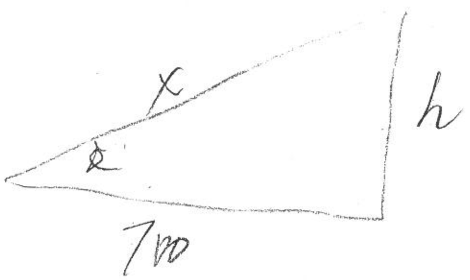
$13y = 7x$

$13 \frac{dy}{dt} = 7 \frac{dx}{dt}$

$13 \frac{dy}{dt} = 7(5)$

$\boxed{\frac{dy}{dt} = \frac{35}{13} \text{ fps}}$

6. An observer stands 700 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 900 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 2400 ft from the ground? How fast is the angle of elevation from the observer to the rocket changing when the rocket is 2700 feet above the ground?



$700^2 + h^2 = x^2$

$2h \frac{dh}{dt} = 2x \frac{dx}{dt}$

$G: \frac{dh}{dt} = 900 \text{ ft/sec}$

$W: \frac{dx}{dt}$ when $h = 2400$

$2(2400)(900) = 2(2500) \frac{dx}{dt}$

$\boxed{\frac{dx}{dt} = 864 \text{ ft/sec}}$

$\tan\theta = \frac{h}{700}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{700} \frac{dh}{dt}$

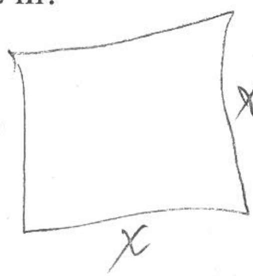
$\frac{d\theta}{dt} = \frac{1}{700}(900)\cos^2\theta$

$\tan\theta = \frac{2400}{700}$

$\theta = 1.28$

$\boxed{\frac{d\theta}{dt} = .1008 \text{ rad/sec}}$

The sides of a square are increasing at 3 in/min. How quickly is the area changing when each side is 12 in?



G: $\frac{dx}{dt} = 3 \text{ in/min}$

$A = x^2$

W: $\frac{dA}{dt}$ when $x = 12$

$\frac{dA}{dt} = 2x\frac{dx}{dt}$

$\frac{dA}{dt} = 2(12)(3) = \boxed{72 \text{ in}^2/\text{min}}$

The volume of a cube is increasing at 14 ft/hour. How quickly is the length of each side increasing when the length of each side is 2 feet?

$V = x^3$

G: $\frac{dV}{dt} = 14 \text{ ft}^3/\text{hr}$

W: $\frac{dv}{dt}$ when $x = 2$

$V = x^3$

$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$

$14 = 3(2)^2\frac{dx}{dt}$

$\boxed{\frac{dx}{dt} = 1.167 \text{ in/hr.}}$

A spherical balloon is being filled with helium at 10 cm/min. Find the rate at which the radius is changing when the radius is 8 cm.

G: $\frac{dV}{dt} = 10 \text{ cm}^3/\text{min}$

W: $\frac{dr}{dt}$ when $r = 8$

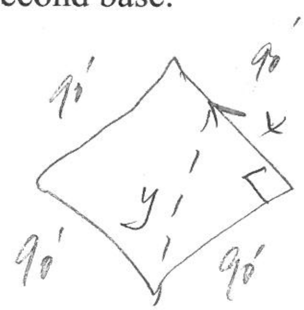
$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

$10 = 4\pi(8)^2\frac{dr}{dt}$

$\boxed{.012 \text{ cm/min} = \frac{dr}{dt}}$

A baseball player is running from first to second base at 10 feet per second. Find the rate at which the distance from home plate to the runner is changing when he is half way to second base.



G: $\frac{dx}{dt} = 10 \text{ ft/sec}$

W: $\frac{dy}{dt}$ when $x = 45$

$90^2 + x^2 = y^2$

$\cancel{2}x\frac{dx}{dt} = \cancel{2}y\frac{dy}{dt}$

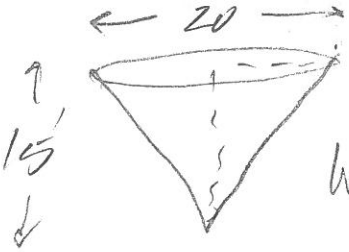
$90^2 + 45^2 = y^2$

$\sqrt{10125} = y$

$\frac{45(10)}{\sqrt{10125}} = \frac{dy}{dt} \rightarrow \boxed{\cancel{14.056 \text{ ft/sec}}}$

$\boxed{4.472 \text{ ft/sec}}$

An inverted conical tank measures 20 feet across the top and 15 feet tall. It is being filled at 10 ft3/min. How quickly is the depth of the water changing when it is 5 feet deep?



G: $\frac{dV}{dt} = 10\ ft^3/min$

W: $\frac{dh}{dt}$ when $h=5$

$V = \frac{1}{3}\pi r^2 h$

$\boxed{.286\ ft/min}$

$\frac{h}{r} = \frac{15}{10}$ $10h = 15r$ $\frac{2}{3}h = r$

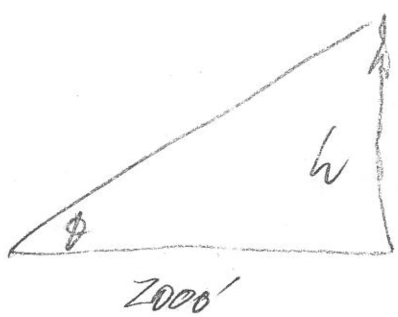
$V = \frac{1}{3}\pi \left(\frac{2}{3}h\right)^2 h$

$V = \frac{4}{27}\pi h^3$ $\frac{dV}{dt} = \frac{4}{9}\pi h^2 \frac{dh}{dt}$

$10 = \frac{4}{9}\pi (5)^2 \frac{dh}{dt}$

A rocket lifts off from a launch pad 2,000 feet from an observation platform. If the rocket's height is given by the equation h = 50t2, find the rate at which the angle of elevation from the platform to the rocket is changing 10 seconds after liftoff?

A rocket is launched from a NASA research facility such that the height of the rocket, t seconds after liftoff, is modeled by the function $h(t) = 50t^2$. Find the rate of change in the angle of elevation of the rocket from an observation deck (located 2,000 feet from the launch pad) ten seconds after liftoff.



G: $h = 50t^2 \rightarrow \frac{dh}{dt} = 100t$

W: $\frac{d\theta}{dt}$ when $t=10$

$\tan\theta = \frac{h}{2000}$ $\tan\theta = \frac{5000}{2000}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{2000}\frac{dh}{dt}$ $\theta = 1.19\ldots$

$\frac{d\theta}{dt} = \frac{1}{2000} 100(10)(\cos\theta)^2$

$\boxed{\frac{d\theta}{dt} = .069\ radians/sec}$