Chapter 1

to find slope

$$\lim_{x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

find area
of each "□"
+ add together

indeterminate form

$\frac{0}{0}$ cannot determine anything

USE ONE OF THESE →

- TRIG ID's
- COPY, DOT, FLOP
- FACTOR + CANCEL
- CONJUGATE

evaluating limits analytic

- plug in numbers "c"
- use conjugate if $\sqrt{\ }$
- find common denominator if fraction

$\boxed{\text{limit of a product is equal to the product of the limits}}$

one sided limits → will always exist
\# or ∄ ∞

- may be infinite
- two sided limits HAVE to be some finite number

continuity: can trace left to right w/ out lifting your pencil

to prove continuity @ $x=c$ you must show

1. $f(c)$ must exist
2. the $\lim_{x \to c} f(x)$ must exist (2 sided limit)
3. $f(c) = \lim_{x \to c} f(x)$

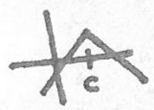
discontinuity: types

- nonremovable → limit does not exist
- removable → hole (limit exists)

(Chapter 2)

differentiability @ x=c

- means that the curve has numeric slope @ x=c ← one value
- a curve will not be differentiable @ x=c if it's discontinuous
  - → sharp turn in the graph



  - → vertical tangent line

- to prove diff. use alternate form of a derivative

$$\lim_{x\to c} \frac{f(x)-f(c)}{x-c}$$

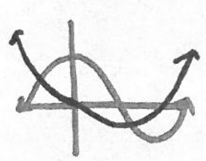
- two sided limit
- you must check cont. first

DNE: not diff

Sketching derivatives

- if f(x) slants uphill f'(x) is above the x axis
- if f(x) slants downhill f'(x) is below the x axis

NOT THE LOCATION



differentiation rules

← $\frac{d}{dx}$: derive the following w respect to x

1. constant rule $\frac{d}{dx} c = 0$
2. power rule $\frac{d}{dx} x^n = nx^{n-1}$
3. constant multiple rule
   $\frac{d}{dx} c\cdot f(x) = c\cdot f'(x)$
4. sum/difference rule
   $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

$y = \sin x$ $y' = \cos x$	$y = \cot x$ $y' = -\csc^2 x$
$y = \cos x$ $y' = -\sin x$	$y = \csc x$ $y' = -\csc x \cot x$
$y = \tan x$ $y' = \sec^2 x$	
$y = \sec x$ $y = \sec x \cdot \tan x$	

projectile motion

meters → $p(t) = -4.9t^2 + v_0t + s_0$

feet → $-16t^2 + v_0t + s_0$

$p(t)$: height above ground after "t" seconds

v_0 → initial velocity

s_0 → initial height

Sign sensitive → +: up −: down

product + quotient rule

$$\frac{d}{dx}[f(x)\cdot g(x)] = f'(x)g(x) + g'(x)f(x)$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

chain rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x))\,g'(x)$$

→ derive outside in

implicit differentiation

1. derive both w respect to x ($\frac{dy}{dx}$) → same rules apply for x's + y's
2. follow any 'y' derivative with $\frac{dy}{dx}$

related rates

1. identify all quantities
   want: given:
2. relate variables w equation
3. derive both sides w respect to time
4. substitute + solve

Chapter 3

absolute extrema

- highest + lowest points on a graph over a given interval
- may occur @
  - end points
  - hill tops / valleys

$f'(x) = 0$ \
$f'(c) =$ undefined } critical #s: values that make slope 0 or undefined

mean value theorem

- must be differentiable!
  - → open interval (a,b)
    - NOT NECCESARILY @ END POINTS
- must be continuous
  - → closed interval $[a,b]$

↑ @ end points!

- ordered pair of the endpoints: in original equation

$m: \frac{y_2 - y_1}{x_2 - x_1}$ ← set equal to derivative

Rolle's Thm

- where the slope = 0
- must be cont + diff.
  - see mean value thm

Intervals of increase + decrease

- x axis intervals

1. find critical #s
2. place on a # line + test in between

+ | − | +

↑ max ↑ min

concavity

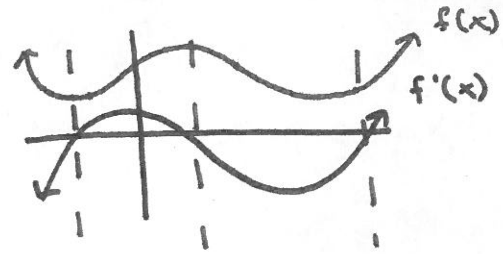
- DOUBLE DERIVATIVE
- if $f''(x) < 0$ then $f(x)$ is concave down ($f''(x)$ = negative)
- if $f''(x) > 0$ then $f(x)$ is concave up ($f''(x)$ = positive)

+ | − | +

↑ ccd ↑ ccu

- inflection pts @ $f''(x) = 0$ or und. ! MAYBE

Sketching f(x) from f'(x)



plug in zeros

doesn't matter where it is!

when $f'(x)$ is below x axis $f(x)$ will be going down, & above axis it will be going up

Slant asymptote → divide + ignore remainder

ex:

$$\begin{array}{r} 2x-1 \\ x-2\overline{)2x^2-5x+5} \\ 2x^2-4x \\ \hline -x+5 \\ -x+2 \\ \hline \cancel{3} \end{array}$$

end behavior

- top heavy: $\frac{x^3 - 2}{x^2 + 3}$ $\boxed{\pm\infty}$
- bottom heavy: $\frac{x^2 + 5}{x^3 - 3}$ $\boxed{0}$
- equally heavy: $\frac{x^2 + 2}{x^2 + 5}$ $\boxed{\text{coefficients}}$

↑ 1

Summary of curve Sketching

intervals	test pt	f(x)	f'(x)	f''(x)	conc

then draw graph!

Chapter 3 (cont.)

optimization (min/max probs)

1. determine equation to be optomized (min or maxed) $P = x \cdot y$
2. if you have more than one variable, use a second equation to eliminate one

   $100 = x + 2y \qquad P = (100 - 2y) \cdot y$

   $100 - 2y = x$
3. find critical #s

   $P' = 100 - 4y \qquad 0 = 100 - 4y$

   $\boxed{y = 25}$
4. prove that 'c' is a min or a max

   $+ \quad | \quad -$

   25
5. find second #

   $\boxed{y = 25} \qquad 100 = x + 2(25)$

   $100 = x + 50$

   $\boxed{x = 50}$

Newton's Meathod

point on f(x)	slope	equation	x=0 @

differential approximation

· differential form of a derivative

$y = x^2 + 3x - 5$

$\frac{dy}{dx} = 2x + 3$

$dy = (2x + 3)\,dx$

when Δx is small $dx \approx \Delta x$ + $\Delta y \approx dy$

$\sqrt{16.1} \approx dy + \sqrt{16} \qquad y = x^{1/2} \quad 16 \quad .1 \qquad dy = \frac{1}{2(4)} \cdot .1 \qquad 4 + .0125 = 4.0125$

$dy = \frac{1}{2} x^{-1/2}\,dx \rightarrow dy = .0125$

dy: propogated error

% error $= \left(\frac{\text{propogated error}}{\text{actual}}\right) \times 100$