Calculus
Mr. Carver

Chapter Five Supplemental Problems

1. Differentiate and then simplify completely:

$f(x) = \arcsin\sqrt{1-64x^2}$

2. Find $\frac{dy}{dx}$:

a) $y = x^{2x-1}$

b) $y = x7^{-3x}$

c) $y = \ln\frac{(x^2+1)^{\frac{1}{2}}}{x(2x^3-1)^2}$

3. Integrate:

a) $\int e^{-x}\tan(e^{-x})dx$

b) $\int\frac{3-e^{3x}}{e^{4x}}dx$

c) $\int(\csc 2\theta - \cot 2\theta)^2\, d\theta$

d) $\int\frac{2-x}{\sqrt{4-x^2}}dx$

e) $\int\frac{1}{x\sqrt{4x^2-1}}dx$

4. The half-life of a Slim Jim is 4,570days. Assuming that you can only eat a maximum of .87 ounces of Slim Jim material, how many days would you have to wait for an 11oz package to be reduced to the appropriate weight? Also assume that you always eat all of the Slim Jim material in the package.

Supplement Problems:

1) $\frac{1}{\sqrt{1-(\sqrt{1-64x^2})^2}} \cdot \frac{1}{2}(1-64x^2)^{-1/2}(-128x)$

$\frac{1}{\sqrt{64x^2}} \cdot \frac{1}{2}(1-64x^2)^{-1/2}(-128x)$

$\frac{-128x}{1 \cdot x\sqrt{1-64x^2}}$

$\frac{-8}{\sqrt{1-64x^2}}$

2) $\ln y = (2x-1)\ln x$

$\frac{1}{y}\frac{dy}{dx} = 2\ln x + (2x-1)\frac{1}{x}$

$\frac{dy}{dx} = x^{(2x-1)}\left[2\ln x + \frac{2x-1}{x}\right]$

3) $y' = 7^{-3x} + x\ln 7\, 7^{-3x}(-3)$

4) $y = \frac{1}{2}\ln(x^2+1) - \left[\ln x + 2\ln(2x^3-1)\right]$

$y' = \frac{1}{2}\left(\frac{1}{x^2+1}\right)(2x) - \left[\left(\frac{1}{x}\right)(1) + 2\left(\frac{1}{2x^3-1}\right)(6x^2)\right]$

5) $a\int e^{-x}\tan e^{-x}\,dx$ $\quad v = e^{-x}$

$-dv = e^{-x}dx$

$-\int \tan v\,dv = \boxed{\ln|\cos e^{-x}| + c}$

6) $\int \frac{3-e^{3x}}{e^{4x}}\,dx = 3\int e^{-4x}dx - \int e^{-x}dx$

$v = -4x$ $\quad v = -x$

$-\frac{1}{4}dv = dx$ $\quad -dv = dx$

$\boxed{\frac{-3}{4}e^{-4x} + e^{-x}dx + c}$

c.) $\int \csc^2 2\theta - 2\csc 2\theta \cot 2\theta + \cot^2 2\theta \, d\theta$

Replace with $\csc^2 2\theta - 1$

$\int 2\csc^2 2\theta - 2\csc 2\theta \cot 2\theta - 1 \, d\theta$

$u = 2\theta \qquad u = 2\theta$

$du = 2\,d\theta \qquad du = 2\,d\theta$

$$-\cot 2\theta + \csc 2\theta - \theta + c$$

d.) $\int \frac{2 - x^2 + 2}{\sqrt{4-x^2}} \, dx$

$\int \frac{-x}{\sqrt{4-x^2}} dx + 2\int \frac{1}{\sqrt{4-x^2}} dx$

$u = 4 - x^2$

$du = -2x\,dx$

$\frac{1}{2}du = -x\,dx$

$2\int \frac{1}{\sqrt{2^2 - x^2}} dx$

$u = x \quad a = 2$

$du = dx$

$\frac{1}{2}\int \frac{1}{u^{1/2}} du + 2\int \frac{1}{\sqrt{a^2 - u^2}} dx$

$\frac{1}{2} 2u^{1/2} + 2\sin^{-1}\frac{x}{2} + c$

$$(4-x^2)^{1/2} + 2\sin^{-1}\frac{x}{2} + c$$

e.) $\int \frac{1}{x\sqrt{(2x)^2-1^2}}\,dx$ $\quad u=2x \quad a=1$

$du = 2\,dx$

~~$\frac{1}{2}du = dx$~~

$= \int \frac{2}{2x\sqrt{(2x)^2-1^2}}\,dx$

$\frac{1}{1}\sec^{-1}2x + c$

4.) $(0, 11)\ (4570, 5.5)$

$a = 11$

$5.5 = 11(b)^{4570}$

$b = .9999$

$y = 11(.9999)^x$

$.87 = 11(.9999)^x$

17,690

$x =$ [redacted] days

$\approx$ 70 years

17,388.695

16,727.77