Use differentials to approximate $\sqrt{4.9}$.

(a) 2.225 (b) 2.250 (c) 2.214

(d) 2.450 (e) None of these

The measurement of the edge of a piece of square floor tile is found to be 12 inches with a possible error of 0.02 inches.

a. Use differentials to approximate the maximum possible error in the area of the tile.

b. Use the answer from part a to estimate the relative error.

c. Use the answer from part b to estimate the percentage error.

The measurement of the circumference of a circle is found to be 54 centimeters. Approximate the percentage error in computing the area of the circle if the possible error in measuring the circumference is 0.6 centimeters. Round your answer to three decimal places.

$y = x^{1/2}$ $dy = \frac{1}{2\sqrt{x}} dx = \frac{1}{2\sqrt{4}}(.9) = \boxed{.225}$

a.) $y = x^2$ $dy = 2x\,dx$

$= 2(12)(.02)$

$= \pm .48 \text{ in}^2$

b) % error $\left(\frac{.48}{12^2}\right)100 = .333\%$

relative error $= .003$

c)

$A = \pi r^2$ $C = 2\pi r$

$r = \frac{C}{2\pi}$

$A = \pi\left(\frac{C}{2\pi}\right)^2$

$A = \frac{1}{4\pi} C^2$

$dA = \frac{1}{2\pi} C\, dC$

$dA = \frac{1}{2\pi}(54)(.06)$

$dA = .51566$ $\frac{}{232.04\ldots} = .222$ %

A FEW GOOD PROBLEMS

From the very mind of TF

On the subject of differentials...

1. The radius of a circle is increased from 2.00 to 2.02 inches
   a. Estimate the resulting change in area
   b. What is the approximate area.
   c. Express the estimate in part a as a % of the original area.

a) $dA = 2\pi r\,dr$

$dA = 2\pi(2)(.02)$

$dA = .251\ in^2$

b.) $4\pi + .251\ in^2$

c.) $100\left(\frac{.251}{4\pi}\right) = 2\%$

2. About how accurately should the radius of a sphere be measured to calculate the surface area $S = 4\pi r^2$ to with in 1 % of its true value.

$\frac{S'}{S}$ $\left(\frac{8\pi r(dr)}{4\pi r^2}\right)100 < 1$

$< .01r$

$\boxed{dr < .005r}$

On the subject of largest and smallest...

3. Hard-Body iron works has contracted to build a 500 ft^3, square-based, open-topped, rectangular steel holding tank for a paper company. The tank is to be made by welding half-inch-thick stainless steel plates together. Your job as the supervising engineer is to find the dimensions that will make the tank weigh as little as possible.

$SA = x^2 + 4xy$ MIN!

$500 = x^2 y$

$y = \frac{500}{x^2}$

$S(x) = x^2 + 4x\left(\frac{500}{x^2}\right)$

$S(x) = x^2 + \frac{2000}{x}$

$S'(x) = 2x - \frac{2000}{x^2}$

$S'(x) = \frac{2x^3 - 2000}{x^2}$

CN's $x = 10$

$S'(9) < 0$ $S'(20) > 0$

10

$x = 10$ $y = 5$