Pt I

1.) $-2\cot\left(\frac{x}{2}\right)+c$

2) $-\frac{1}{2\cot^2 x}+$

3) $-\frac{2}{3}\cot^{3/2}(x)+c$

4.) $\ln|\sec x-1|+c$

5) $\ln|1+\cos x|+c$

6.) $0+c$

7) $e^{\sec x}+c$

8.) $\int \sin^2(2x)+2\sin(2x)\cos(2x)+\cos^2(2x)\,dx$

$\int 1+2\sin(2x)\cos(2x)$ $\quad u=\sin(2x)$

$x+\frac{1}{2}\sin^2(2x)+c$ $\quad \frac{1}{2}du=2\cos(2x)$

Pt II

1.) $\int\frac{x}{\sqrt{4-x^2}}dx+2\int\frac{1}{\sqrt{4-x^2}}dx$

$\frac{1}{2}\int u$

$\boxed{-\sqrt{4-x^2}+2\sin^{-1}\left(\frac{x}{2}\right)+c}$

2) $x^2+0x+4\overline{\smash{)}3x^3+0x^2+0x-2}$ $\quad$ quotient $3x$

$\underline{3x^3+0x^2+12x}$

$-12x-2$

$\int 3x+\frac{-12x}{x^2+4}-\frac{2}{x^2-4}$

$\boxed{\frac{3}{2}x^2-6\ln|x^2+4|-\tan^{-1}\left(\frac{x}{2}\right)+c}$

3.) $a=1$ $v=3x$ $\frac{1}{3}dv=3\,dx$

$\frac{1}{3}\int\frac{1}{\sqrt{a^2-v^2}}\,dv$ $\qquad \frac{1}{3}\sin^{-1}(3x)\Big]_0^{1/6}$

$$\boxed{\frac{\pi}{9}-0}$$

4.) $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)\Big]_{\sqrt{3}}^{3}$ → $\frac{\pi}{12}-\frac{\pi}{18}=\boxed{\frac{\pi}{36}}$

5.) $a=1$ $v=t^2$ $\frac{1}{2}dv=2t$ $\qquad \frac{1}{2}\int\frac{1}{\sqrt{a^2-v^2}}\,dv$

$$\boxed{\frac{1}{2}\sin^{-1}(t^2)+c}$$

6.) $a=1$ $v=x+1$ $\qquad \boxed{\sin^{-1}(x+1)+c}$

7.) $\boxed{\frac{1}{3}\tan^{-1}\left(\frac{x-3}{3}\right)+c}$

8.) $-(x^2+4x+4)+4$ $\qquad \int\frac{1}{\sqrt{2^2-(x+2)^2}}\,dx$ $\qquad a=2$ $v=x+2$

$dv=dx$

$$\boxed{\sin^{-1}\left(\frac{x+2}{2}\right)+c}$$

9.) $\int\frac{x+2}{\sqrt{-x^2-4x}}$ $\qquad -\frac{1}{2}\int v^{-1/2}\,dv$

$$\boxed{-\sqrt{-x^2-4x}+c}$$

Pt. #

1.) $\frac{1}{6}\sin^3(2x)+c$

2.) $\int \cos^2(2x)\left(1-\sin^2(2x)\right)^2\sin(2x)$

$\int \cos^2(2x)\left(1-2\sin^2(2x)+\sin^4(2x)\right)\sin(2x)\,dx$

$\int \cos^2(2x)\sin(2x)\,dx-2\int\sin^3(2x)\cos(2x)\,dx+\int\sin^5(2x)\cos(2x)\,dx$

$v=\cos(2x)$

$\frac{1}{2}\,dv=-\sin(2x)\,dx$

$-\frac{1}{2}\int v^2\,dv+\int v^3\,dv-\frac{1}{2}\int v^5\,dv$

$\boxed{-\frac{1}{6}\cos^3(2x)+\frac{1}{4}\cos^4(2x)-\frac{1}{12}\cos^6(2x)+c}$

3) $\int\left(1-\sin^2\left(\frac{x}{3}\right)\right)\cos\left(\frac{x}{3}\right)dx=\int\cos\left(\frac{x}{3}\right)dx-\int\sin^2\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)dx$

$v=\frac{x}{3}$ $\quad$ $\ddot{v}=\sin\left(\frac{x}{3}\right)$

$3\,dv=dx$ $\quad$ $3\,d\ddot{v}=\cos\left(\frac{x}{3}\right)dx$

$=3\int\cos v\,dv-3\int\ddot{v}^2\,d\ddot{v}$

$\boxed{3\sin\left(\frac{x}{3}\right)-\sin^3\left(\frac{x}{3}\right)+c}$

4.) $\frac{1}{4}\int\left(1-\cos(4x)\right)^2=\frac{1}{4}\int 1-2\cos(4x)+\cos^2(4x)\,dx$

$\frac{1}{4}\int 1\,dx-\frac{1}{2}\int\cos(4x)\,dx+\frac{1}{4}\int\frac{1+\cos(8x)}{2}\,dx$

$\frac{1}{4}\int 1\,dx-\frac{1}{2}\int\cos(4x)\,dx+\frac{1}{8}\int 1\,dx+\frac{1}{8}\int\cos(8x)\,dx$

$v=4x$ $\quad$ $v=8x$

$\frac{1}{4}\,dv=dx$ $\quad$ $\frac{1}{8}\,dv=dx$

$\frac{1}{4}\int 1\,dx-\frac{1}{8}\int\cos v\,dv+\frac{1}{8}\int 1\,dx+\frac{1}{64}\int\cos v\,dv$

$\boxed{\frac{1}{4}x-\frac{1}{8}\sin(4x)+\frac{1}{8}x+\frac{1}{64}\sin(8x)+c}$

5.) $\int \sin^4 x \cos^2 x \, dx = \int \left(\left(\frac{1-\cos(2x)}{2}\right)\right)^2 \left(\frac{1+\cos(2x)}{2}\right) dx$

	1	$-2\cos(2x)$	$\cos^2(2x)$
1	1	$-2\cos(2x)$	$\cos^2(2x)$
$\cos(2x)$	$\cos(2x)$	$-2\cos^2(2x)$	$\cos^3(2x)$

$\frac{1}{8}\int 1 - \cos(2x) - \cos^2(2x) + \cos^3(2x) \, dx$

$\frac{1}{8}\int 1 \, dx - \frac{1}{8}\int \cos(2x) - \frac{1}{8}\int \frac{1+\cos(4x)}{2} dx + \frac{1}{8}\int (1-\sin^2(2x))\cos(2x) \, dx$

$\underbrace{\frac{1}{8}\int 1 dx - \cancel{\frac{1}{8}\int \cos(2x) dx} - \frac{1}{16}\int 1 \, dx}_{} - \frac{1}{16}\int \cos(4x) \, dx + \cancel{\frac{1}{8}\int \cos(2x)} - \frac{1}{8}\int \sin^2(2x)\cos(2x)$

$\frac{1}{16}\int 1 \, dx - \frac{1}{16}\int \cos(4x) \, dx - \frac{1}{8}\int \sin^2(2x)\cos(2x) \, dx$

$u = 4x \quad \frac{1}{4} du = dx \qquad u = \sin(2x) \quad \frac{1}{2} du = \cos(2x) \, dx$

$$\boxed{\frac{1}{16} x - \frac{1}{64}\sin(4x) - \frac{1}{48}\sin^3(2x) + c}$$

6.) $\int \sec^4(5x) \, dx = \int (\tan^2(5x) + 1) \sec^2(5x) \, dx$

$\int \tan^2(5x)\sec^2(5x) + \int \sec^2(5x) \, dx$

$u = \tan(5x) \qquad u = 5x$

$\frac{1}{5} du = \sec^2(5x) \, dx \qquad \frac{1}{5} du = dx$

$$\boxed{\frac{1}{15}\tan^3(5x) - \frac{1}{5}\tan(5x) + c}$$

7) $\int \sec^4(\pi x)\sec(\pi x)\tan(\pi x)\,dx$

$v = \sec(\pi x)$

$\frac{1}{\pi}\,dv = \sec(\pi x)\tan(\pi x)\,dx$

$\frac{1}{\pi}\int v^4\,dv = \boxed{\frac{1}{5\pi}\sec^5(\pi x) + c}$

8) $\int (\sec^2(3x) - 1)\sec(3x)\tan(3x)\,dx$

$\int \sec^2(3x)\sec(3x)\tan(3x)\,dx - \int \sec(3x)\tan(3x)\,dx$

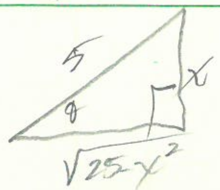
$v = \sec(3x)$ $\quad$ $v = 3x$

$\frac{1}{3}\,dv = \sec(3x)\tan(3x)\,dx$ $\quad$ $\frac{1}{3}\,dv = dx$

$\frac{1}{3}\int v^2\,dv - \frac{1}{3}\int \sec v \tan v\,dv$

$\boxed{\frac{1}{9}\sec^3(3x) - \frac{1}{3}\sec(3x) + c}$

Pt. IV

1) 

$5\sin\theta = x$

$5\cos\theta\, d\theta = dx$

$5\cos\theta = \sqrt{25-x^2}$

$$\int \frac{1}{25\sin^2\theta\ \cancel{5\cos\theta}}\ 5\cos\theta\, d\theta$$

$$\frac{1}{25}\int \csc^2\theta\, d\theta = -\frac{1}{25}\cot\theta + c$$

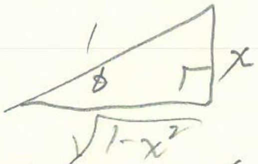
$$\boxed{-\frac{1}{25}\left(\frac{\sqrt{25-x^2}}{x}\right)+c}$$

2) $u = 16-4x^2$

$-\frac{1}{8}\, du = -8x\, dx$

$$-\frac{1}{8}\int u^{1/2}\, du$$

$$\boxed{-\frac{1}{12}(16-4x^2)^{3/2}+c}$$

3) 

$\sin\theta = x$

$\cos\theta\, d\theta = dx$

$\cos\theta = \sqrt{1-x^2}$

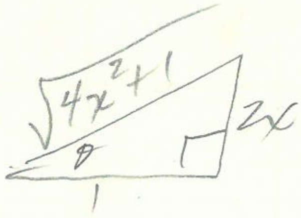
$$\int \frac{\cos\theta}{(\sin\theta)^4}\cos\theta\, d\theta \qquad \int \frac{\cos^2\theta}{\sin^4\theta}\, d\theta$$

$$\int \csc^2\theta\cot^2\theta\, d\theta \qquad u = \cot\theta$$

$-du = +\csc^2\theta\, d\theta$

$$-\int u^2\, du = -\frac{1}{3}(\cot\theta)^3 + c$$

$$\boxed{-\frac{1}{3}\left(\frac{\sqrt{1-x^2}}{x}\right)^3 + c}$$

4) 

$\tan\theta = 2x$

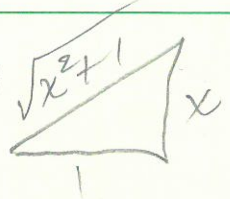
$\frac{1}{2}\sec^2\theta\, d\theta = dx$

$\sec\theta = \sqrt{4x^2+1}$

$$\frac{1}{2}\int \frac{1}{\sec\theta}\sec^2\theta\, d\theta$$

$$= \frac{1}{2}\ln|\sec\theta + \tan\theta| + c = \boxed{\frac{1}{2}\left|\sqrt{4x^2+1} + 2x\right| + c}$$

5.)



$\int (\sec\theta)^{-3} \sec^2\theta \, d\theta$

$\tan\theta = x$

$\sec^2\theta \, d\theta = dx$

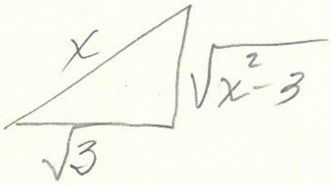
$\sec\theta = \sqrt{x^2+1}$

$\int \cos\theta \, d\theta$

$\sin\theta + c$

$$\boxed{\frac{x}{\sqrt{x^2+1}} + c}$$

6.)



$\sqrt{3}\sec\theta = x$

$\sqrt{3}\sec\theta\tan\theta \, d\theta = dx$

$3\tan\theta = \sqrt{x^2-3}$

$$\int \frac{3\tan\theta}{\cancel{\sqrt{3}\sec\theta}} \cancel{\sqrt{3}\sec\theta} \tan\theta \, d\theta$$

$3\int \tan^2\theta \, d\theta$

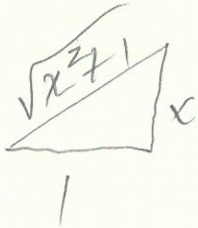
$3\int \sec^2\theta - 1 \, d\theta$

$$3\tan\theta - 3\theta \Big] \quad \left. \frac{\sqrt{x^2-3}}{\sqrt{3}} - 3\sec^{-1}\frac{x}{\sqrt{3}} \right]_{\sqrt{3}}^{2}$$

$$\left(\frac{1}{\sqrt{3}} - \frac{\pi}{2}\right) - \left(0 - 0 \right)$$

$$\boxed{\frac{2 - \sqrt{3}\,\pi}{2\sqrt{3}}}$$

7.)



$\tan\theta = x$

$\sec^2\theta \, d\theta = dx$

$\sec\theta = \sqrt{x^2+1}$

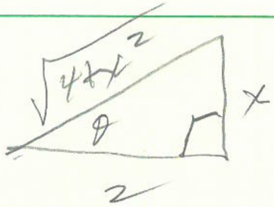
$$\int \frac{1}{\sec^4\theta} \cancel{\sec^2\theta} \, d\theta$$

$\int \cos^2\theta \, d\theta$

$$\frac{1+\cos(2\theta)}{2} = \int \frac{1}{2} d\theta + \frac{1}{2}\int \cos(2\theta)$$

$$\boxed{\frac{1}{2}\tan^{-1}x + \frac{1}{2}\frac{x}{\sqrt{x^2+1}} \frac{1}{\sqrt{x^2+1}} + c}$$

8) $\int \frac{x^2}{\left(\sqrt{4+x^2}\right)^4}$



$$2\tan\theta = x$$

$$2\sec^2\theta \, d\theta = dx$$

$$2\sec\theta = \sqrt{4+x^2}$$

$$\int \frac{4\tan^2\theta}{16\sec^4\theta} 2\sec^2\theta \, d\theta = \frac{1}{2}\int \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} = \frac{1}{2}\int \sin^2\theta \, d\theta$$

$$\int \frac{1}{2}\left[\frac{1-\cos(2\theta)}{2}\right] d\theta = \frac{1}{4}\int 1-\cos(2\theta)\, d\theta$$

$$\boxed{= \frac{1}{4}\tan^{-1}\left(\frac{x}{2}\right) - \frac{1}{8}\underset{\hookrightarrow 2\sin\theta\cos\theta}{\sin(2\theta)} + c}$$

$$\frac{1}{4}\tan^{-1}\left(\frac{x}{2}\right) - \frac{1}{8}(2)\frac{x}{\sqrt{4+x^2}}\frac{2}{\sqrt{4+x^2}} + c$$

Pt V

1) $\int \frac{1}{x^2-5x+6}\,dx = \frac{A}{(x-3)} + \frac{B}{(x-2)}$

$A(x-2) + B(x-3) = 1$

$x=2 \quad B=-1$

$x=3 \quad A=1$

$= \int \frac{1}{x-3}\,dx - \int \frac{1}{x-2}\,dx$

$\boxed{= \ln|x-3| - \ln|x-2| + c}$

2) $\int \frac{5x^2+20x+6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$5x^2+20x+6 = A(x^2+2x+1) + Bx(x+1) + Cx$

$5x^2+20x+6 = (A+B)x^2 + (2A+B+C)x + A$

$A=6 \quad A+B=5 \quad 12-1+C=20$

$B=-1 \quad C=9$

$= \int \frac{6}{x} + \int \frac{-1}{x+1} + \int \frac{9}{(x+1)^2}\,dx$

$\boxed{6\ln x - \ln|x+1| - \frac{9}{(x+1)} + c}$

3) $\int \frac{2x^3-4x-8}{x(x-1)(x^2+4)}\,dx = \frac{A}{x} + \frac{B}{(x-1)} + \frac{Cx+D}{x^2+4}$

	x	-1
x^2	x^3	$-x^2$
4	$4x$	-4

$2x^3-4x-8 = A(x^3-x^2+4x-4) + B(x^3+4x) + (Cx+D)(x^2-x)$

$= Ax^3 - Ax^2 + 4Ax - 4A + Bx^3 + 4Bx + Cx^3 - Cx^2 + Dx^2 - Dx$

$2x^3-4x-8 = (A+B+C)x^3 + (-A-C+D)x^2 + (4A+4B-D)x - 4A$

$-4A=-8$

$\boxed{A=2}$

$A+B+C=3$

$-A-C+D=0$

$4A+4B-D=-4$

$B+C=1 \quad C=1-B$

$-C+D=2$

$4B+D=-12 \quad D=-12-4B$

$\boxed{D=8}$

$-1+B-12-4B=2$

$-3B=15$

$\boxed{B=-5}$

$\boxed{C=6}$

$$3(\text{CONT}) = \int \frac{2}{x} + \int \frac{-5}{x-1} + \int \frac{6x+8}{x^2+4}\,dx$$

$$6\int \frac{x}{x^2+4} + 8\int \frac{1}{x^2+4}\,dx \quad u=x \quad a=2 \quad du=dx$$

$u=x^2+4$
$\frac{1}{2}du=dx$

$$\boxed{= 2\ln|x| - 5\ln|x-1| + 3\ln|x^2+4| + 4\tan^{-1}\left(\frac{x}{2}\right) + c}$$

④ OMIT

5. $\int \frac{3}{(x+2)(x-1)}\,dx = \frac{A}{x+2} + \frac{B}{x-1}$ $\quad A(x-1) + B(x+2) = 3$

$\boxed{-\ln|x+2| + \ln|x-1| + c}$ $\quad x=1 \quad B=1, \quad x=-2 \quad A=-1$

6.) $\int \frac{\cancel{x+1}}{(x+3)\cancel{(x+1)}} = \frac{A}{x+3} + \frac{B}{x+1} \rightarrow x+1 = A(x+1) + B(x+3)$

↳ $\boxed{\ln|x+3| + c}$

7.) $\int \frac{x+2}{x(x-4)}\,dx = \frac{A}{x} + \frac{B}{x-4} \rightarrow x+2 = A(x-4) + Bx$

$x=4 \quad B=\frac{3}{2}$

$-\frac{1}{2}\int \frac{1}{x}\,dx + \frac{3}{2}\int \frac{1}{x-4}$ $\quad x=0 \quad A=-\frac{1}{2}$

$\boxed{-\frac{1}{2}\ln|x| + \frac{3}{2}\ln|x-4| + c}$

8.) ↘

8.) $x^2+x-2 \,\overline{\big)\, x^3+0x^2-x+3}$ with quotient $x-1$

$$-(x^3+x^2-2x)$$

$$-x^2+x+3$$

$$-(-x^2+x+2)$$

$$1$$

$$= \int x-1+\frac{1}{(x+2)(x-1)}\,dx = \frac{A}{x+2}+\frac{B}{x-1}$$

$1=A(x-1)+B(x+2)$

$x=1 \quad B=1/3$

$x=-2 \quad A=-1/3$

$$\int x-1-\frac{1}{3}\int\frac{1}{x+2}\,dx+\frac{1}{3}\int\frac{1}{x-1}\,dx$$

$$\boxed{\frac{1}{2}x^2-x-\frac{1}{3}\ln|x+2|+\frac{1}{3}\ln|x-1|+c}$$