

Determine the area of the region.

1. $f(y) = y(2 - y), g(y) = -y$

2. $y = x, y = 2 - x, y = 0$

3. $f(x) = \cos x, g(x) = 2 - \cos x, 0 \leq x \leq 2\pi$

For problems 4 – 6, find the volume of the solid formed by revolving the given region about the y -axis.

4. $x = -y^2 + 4y$, above the line $y = 1$

5. $y = 2x^2, y = 0, x = 2$

6. $y = \frac{1}{x^2}, y = 0, x = 1, x = 4$

For problems 7 – 9, find the volume of the solid formed by revolving the given region about the x-axis.

7. $y = \sqrt{4 - x^2}, x = 0, y = 0$

8. Above $y = 2, y = 4 - \frac{x^2}{2}$

9. $x = 12(y^2 - y^3), x = 0$

10. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about $y = 8$:

$$y = 2x^2, y = 0, x = 2$$

11. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about $x = -2$:

$$y = x^2 - 4, y = 4 - x^2$$

12. Find the volume of the solid whose base is bounded by the graphs of $y = x + 1$ and $y = x^2 - 1$, with the indicated cross sections taken perpendicular to the x-axis.

(a) Squares

(b) Rectangles of height 1

(c) RIGHT ISOSCELES TRIANGLES WITH HYPOTENUSE ON
BASE

(d) EQUILATERAL TRIANGLES WITH A CIRCLE OF
RADIUS = 5 FOR A BASE

KEY
 1-5 eliminate
 9 & 11
 For

Determine the area of the region.

1. $f(y) = y(2-y), g(y) = -y$

$$2y - y^2 = -y$$

$$y^2 + 3y = 0$$

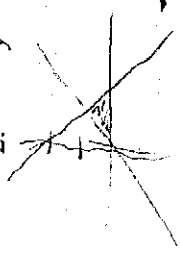
$$y = 0, -3$$

$$\int_{-3}^0 (y^2 + 3y) dy$$

$$\left[\frac{1}{3}y^3 + \frac{3}{2}y^2 \right]_{-3}^0$$

$$(0+0) - \left(-9 + \frac{27}{2} \right) = \boxed{\frac{9}{2}}$$

2. $y = x, y = 2-x, y = 0$

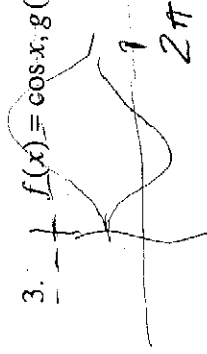


$$\int_0^1 x dx + \int_1^2 (2-x) dx$$

$$\left[\frac{1}{2}x^2 \right]_0^1 + \left[2x - \frac{1}{2}x^2 \right]_1^2$$

$$\frac{1}{2} + 2 - \frac{3}{2} = \boxed{1}$$

3. $f(x) = \cos x, g(x) = 2 - \cos x, 0 \leq x \leq 2\pi$



$$\int_0^{2\pi} (\cos x - 2 + \cos x) dx$$

$$\int_0^{2\pi} (2\cos x - 2) dx$$

$$\left[2\sin x - 2x \right]_0^{2\pi}$$

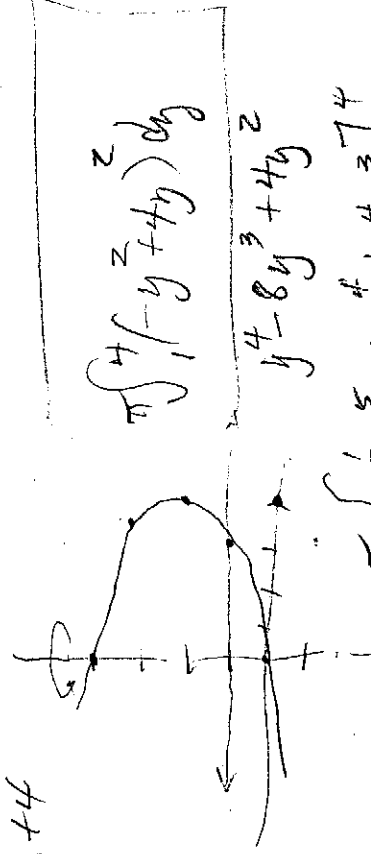
$$(0 - 4\pi) - (0 - 0) = \boxed{4\pi}$$

For problems 4 - 6, find the volume of the solid formed by revolving the given region about the y-axis.

4. $x = -y^2 + 4y$, above the line $y = 1$

$$-(y^2 - 4y + 4) + 4$$

$$-(y-2)^2 + 4$$



$$\pi \int_1^4 (-y^2 + 4y) dy$$

$$\left[-\frac{1}{3}y^3 + 2y^2 \right]_1^4$$

$$\left(-\frac{64}{3} + 32 \right) - \left(-\frac{1}{3} + 2 \right)$$

$$\left(\frac{32}{3} - \frac{5}{3} \right) = \boxed{\frac{27}{3} = 9}$$

$$\pi \left[\frac{1}{5}y^5 - 2y^4 + \frac{4}{3}y^3 \right]_1^4$$

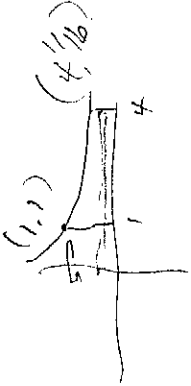
5. $y = 2x^2, y = 0, x = 2$

$$\pi \int_0^8 2 \, dy - \pi \int_0^8 (\sqrt{y/2})^2 \, dy$$

$$4\pi y \Big|_0^8 - \frac{\pi}{4} \frac{1}{2} y^2 \Big|_0^8$$

$$32\pi - 8\pi = 24\pi$$

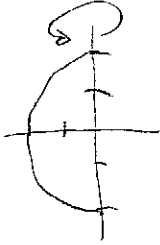
6. $y = \frac{1}{x^2}, y = 0, x = 1, x = 4$



$$\pi \int_0^{1/16} 4 \, dx - \pi \int_0^{1/16} x^2 \, dx + \int_{1/16}^1 (\sqrt{\frac{1}{y}})^2 \, dy - \int_{1/16}^1 1^2 \, dy$$

For problems 7 - 9, find the volume of the solid formed by revolving the given region about the x-axis.

7. $y = \sqrt{4-x^2}, x = 0, y = 0$

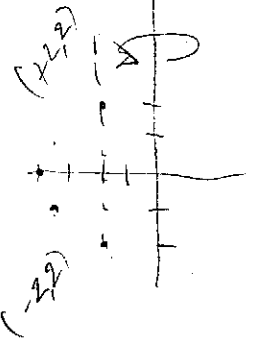


$$\pi \int_{-2}^2 (\sqrt{4-x^2})^2 \, dx = \pi [4x - \frac{1}{3}x^3]_{-2}^2$$

$$= \pi \left[\left(8\pi - \frac{8}{3}\pi \right) - \left(-8\pi + \frac{8}{3}\pi \right) \right]$$

$$= \frac{16}{3}\pi - \left(-\frac{16}{3}\pi \right) = \frac{32}{3}\pi$$

8. Above $y = 2, y = 4 - \frac{x^2}{2}$



$$\pi \int_{-2}^2 \left(4 - \frac{x^2}{2} \right)^2 \, dx - \pi \int_{-2}^2 (2)^2 \, dx$$

$$= \pi \left[\int_{-2}^2 \left(16 - 4x^2 + \frac{1}{4}x^4 \right) \, dx - \int_{-2}^2 4 \, dx \right]$$

$$= 12x - \frac{4}{3}x^3 + \frac{1}{20}x^5 \Big|_{-2}^2 - 2x \Big|_{-2}^2$$

$$= 24 - \frac{32}{3} + \frac{16}{5} - 4 = 20 - \frac{32}{3} + \frac{16}{5} = 29.024$$

Shell $y^2(y-1), y=0, y=1$ 44

~~Shell~~
 $x = 12(y^2 - y^3), x=0, x = 12(y^2 - y^3), x=0$

$$2\pi \int_0^1 x (12(x^2 - x^3)) dx$$

$$24\pi \int_0^1 x^3 - x^4 dx$$

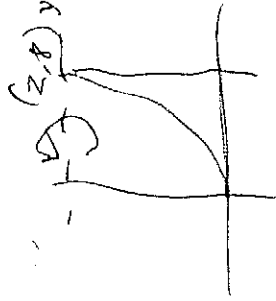
$$24\pi \left[\frac{1}{4}x^4 - \frac{1}{5}x^5 \right]_0^1$$

$$24\pi \left[\frac{1}{20} - 0 \right] = 6\pi/5$$

≈ 3.77

10. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about $y = 8$:

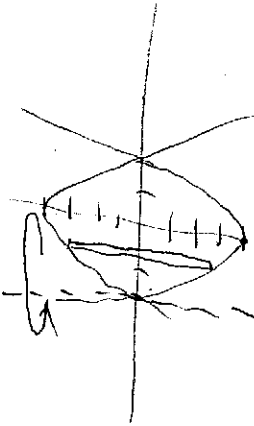
$(2, 8) y = 2x^2, y = 0, x = 2$



$$\pi \int_0^2 8^2 dx - \pi \int_0^2 (8 - 2x^2)^2 dx$$

11. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about $x = -2$:

$y = x^2 - 4, y = 4 - x^2$

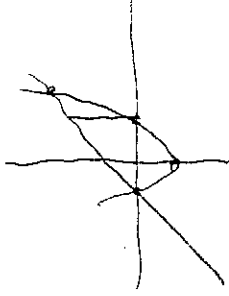


Shell

$$2\pi \int_{-2}^2 x(4 - x^2 - (x^2 - 4)) dx$$

12. Find the volume of the solid whose base is bounded by the graphs of $y = x + 1$ and $y = x^2 - 1$, with the indicated cross sections taken perpendicular to the x-axis.

(a) Squares



$$\int_{-1}^2 (x+1 - (x^2-1))^2 dx$$

(b) Rectangles of height 1

$$\int_{-1}^2 ((x+1) - (x^2-1))(1) dx$$

c) ~~Right~~ Right isosceles triangles w/ leg on base

$$\frac{1}{2} \int_{-1}^2 (x+1 - (x^2-1)) \left(\frac{x+1 - (x^2-1)}{2} \right) dx$$



~~Right isosceles triangles~~

d) ~~Right isosceles triangles~~ Right isosceles triangles w/ leg on base

$$\frac{1}{2} \int_{-1}^2 (x+1 - (x^2-1))^2 dx$$

$$\frac{1}{2} \int_{-1}^2 ((x+1) - (x^2-1)) \sqrt{\frac{(x+1 - (x^2-1))^2}{2}} dx$$